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US Army Electronics Technology and Devices Laboratory (ECOM)

9 MEMORANDUM FILE REPORT

6 THE COPLANAR ELECTRON TUBE

by

12 48p.

10 Mortimer H. Zinn

Advanced Concept and Techniques
Beam, Plasma and Display Technical Area

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ABSTRACT

Coplanar electron tubes consisting of emitting, controlling and collecting electrodes on a single heated plane have been proposed as a method of achieving high temperature and high radiation resistant devices. In order to design these devices, the scaling laws must be obtained. Using computer techniques it was determined that while the plate current of a device still followed a general three-halves power curve, the effect of electrode areas and electrode spacing are far less pronounced then in multiplanar tubes. A number of auxiliary problems were studied and data was obtained indicating that both alumina and beryllia would be suitable substrates for these devices. Single crystal alumina (sapphire) was found to be satisfactory, but polycrystalline alumina was not, indicating some reaction with active material from the cathode. Polycrystalline beryllia, on the other hand, was satisfactory. A trough strip line was developed for use in a distributed amplifier, power coplanar tetrode, but time did not permit, nor did results warrant, the construction of such a device.

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THE COPLANAR ELECTRON TUBE

INTRODUCTION

The coplanar electron tube was proposed as the active element in an integrated vacuum circuit (IVC) by Dore, Geppert, and Mueller while working at Stanford Research Institute. Information on the device was obtained primarily by private communications and finally by reports on the cathode phases of the program.[1] The coplanar tube appeared to have two potential advantages. Small spacings between grid and cathode could be achieved and maintained using photo-lithographic techniques, since the two electrodes are located on the same substrate; and, since the entire device operates at cathode temperature (600 - 700°C), it should provide a device capable of operation at high temperatures, and at the same time have all of the capabilities of a vacuum device with respect to operation under high nuclear radiation exposures. These possible advantages together with the potential of constructing entire circuits on an insulating substrate, an IVC, indicated that the feasibility of the circuit and its potential use in Army equipment should be investigated. Problems to be evaluated were: theoretical performance capabilities of active coplanar devices; limitations due to substrate operation at high temperature; limitations due to thermionic emission capabilities of the photolithographic cathode; limitations due to unwanted thermionic emission from control and collector electrodes on the hot substrate; the performance of a simple triode; and the proper capabilities of devices which could be designed using basic coplanar principles. A program was initiated to investigate these various facets of the coplanar tube design.

BACKGROUND

The thermionic electron tube has evolved considerably since the first triode was constructed by introducing a coarse wire grid between the cathode and the anode of a vacuum diode. The grid was capable of controlling the flow of electron current by a relatively small change in voltage on the grid or control electrode. The current flowing to the anode of a three element tube obeys the Child-Langmuir space charge relationship originally derived by Child[2] for a vacuum diode:

$$I_b = G\,E_d^{\,3/2} \qquad (1)$$

where I_b = the current to the anode

G = a constant designated perveance

E_d = equivalent diode voltage

1. B. Dore, D. Geppert, & R. Mueller, "Low Temperature Thermionic Emitter," NASA Contract No. NAS 12-607.

2. C. D. Child, Phys Rev., 32, pp. 498, (1911).

The equivalent diode voltage is in its simplest approximation:

$$E_d = \mu E_g + E_b \qquad (2)$$

where E_g = Voltage on the grid (negative for equation (1) to be accurate)

μ = Amplification constant

E_b = Voltage on the anode.

The amplification constant can be further defined by combining equations (1) and (2) and differentiating with respect to E_g, holding the plate current constant.

$$0 = \frac{\partial E_b}{\partial E_g} + \mu$$

$$\mu = -\frac{\partial E_b}{\partial E_g} \qquad (3)$$

Another important tube characteristic can be defined as the transconductance, G_m, the change in plate current due to a change in grid voltages, with constant plate voltage or

$$G_m = \frac{\partial I_b}{\partial E_g} = \frac{3}{2}\mu G\,(\mu E_g + E_b)^{1/2}$$

but $$(\mu E_g + E_b)^{1/2} = \frac{I_b}{G}^{1/3}$$

or $$G_m = \frac{3}{2}\mu G\left(\frac{I_b}{G}\right)^{1/3} = \frac{3}{2}\mu(G^2 I_b)^{1/3} \qquad (4)$$

The transconductance is thus a function of the perveance of the tube (which is dependent upon geometrical factors such as cathode and anode area and grid-to-cathode and grid-to-plate spacings) and the current level at which the tube is operating.

The transconductance and the μ of a triode are related to each other through the relationship:

$$\frac{\partial I_b}{\partial E_g} \cdot \frac{\partial E_b}{\partial I_b} = -\frac{\partial E_b}{\partial E_g} \qquad (5)$$

or $$G_m \cdot r_p = \mu \qquad (6)$$

where r_p = plate resistance

Equations (5) and (6) indicate that only two of the three triode characteristics are independent.

Of the three characteristics the transconductance is the most important one with respect to the bulk of applications of the triode as a power amplifier. It is used in a number of derived and "artificial" figures of merit, and such factors as gain and bandwidth (and therefore, gain-bandwidth product) are directly related to this factor. Since transconductance is proportional to the two-thirds power of the perveance constant G, which in turn is directly proportional to the area and inversely proportional to the interelectrode spacings, high transconductance can be achieved by increasing the area of the tube or decreasing the spacings. This cannot, however, be done without affecting the plate current drawn at a given diode voltage. Thus, attempts have been made to improve the transconductance to plate current ratio, which is one of the "artificial" figures of merit of a triode (or tetrode or pentode). The ratio, R, is

$$R = \frac{G_m}{I_b} = \frac{3}{2}\mu\left(\frac{G}{I_b}\right)^{2/3} \tag{7}$$

This ratio decreases with increasing current making it easier to obtain high values at currents approaching cut-off as is acceptable for small signal amplifiers, compared to high peak current levels, such as the peak of the RF, required for power amplifiers. As will be shown later, high levels of R are not an absolute necessity for power amplifiers. Indeed one must settle for a value of $R < R_{max}$ in order to maintain efficiency. The exact design value is a trade-off between tube size and efficiency. It is further limited by the peak current density capability of the cathode. These aspects of the problem have not been discussed in the previous literature.[3,4]

The relationship shown in Equation (1) is based on a rigorous solution of Poisson's equation for a planar diode and similar results can be obtained for both the cylindrical and the spherical diode. There was no basis, however, for the belief that the equation could be applied directly to the coplanar diode. Since a simple conformal transformation could not be used to obtain a recognizable form of geometry for which a solution was available, it was decided to obtain a computer solution to the Poisson equation which could be used to obtain a relationship for scaling purposes. Two computer programs were written, one, by Capt. T. Freeman during his tour of duty at this Laboratory, which uses the conventional electron trajectory approach, and the second by the author using an approach which will be discussed below. Capt. Freeman's program seemed to give reasonable calculations for collected anode current of a coplanar triode. When trajectories were plotted, however, the program showed no electrons landing on the anode.

3. J. R. Pierce, "Theoretical Limitation to Transconductance in Certain Types of Vacuum Tubes," Proc. IRE, 31, pp. 657 (Dec. 1943).

4. G. R. Kilgore, "Beam-deflection Control for Amplifier Tubes," RCA Rev, 8, pp. 480-505 (Sep 1947).

Since debugging of the complex program by someone other than the author was difficult and extending it to a tetrode configuration would be even more difficult, all effort on this version of the program was dropped. A copy of the program is, however, maintained on file.

DISCUSSION

Theoretical Performance of a Coplanar Device

The coplanar triode, using a grid and anode on both sides of a cathode, is shown in Figure 1a. Figure 1b shows a tetrode design.

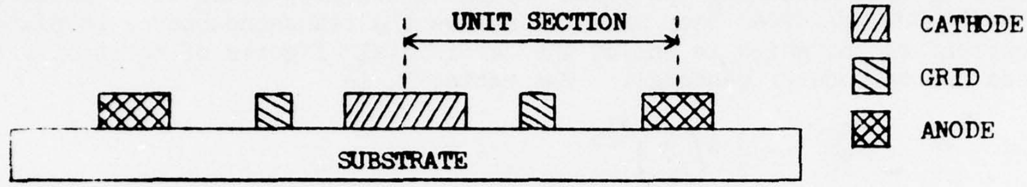


Figure 1a. Coplanar Triode Showing Grid and Anode Sections Surrounding a Central Cathode.

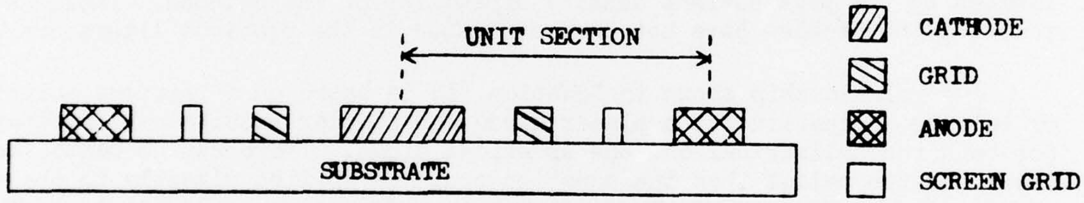


Figure 1b. Coplanar Tetrode Showing Grid, "Screen" Grid, and Anode Sections Surrounding a Central Cathode.

The dashed lines in each figure show the configuration of a unit section, which was used for computer calculation purposes. To obtain the current for any given length of the device shown, the calculated current per unit length would have to be multiplied by the actual length and by a factor of two since there are two equal sections of triode. Actually the number of sections can be iterated indefinitely with the section of anode outside of the dashed line serving as the anode for the next triode section as shown in Figure 2.

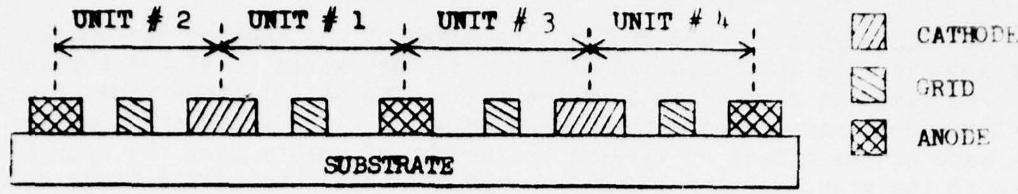


Figure 2. The Coplanar Triode with a Second Cathode Stripe Surrounded by Grid and Anode.

For the configuration shown in Figure 2 there are two additional triode sections or the total current, I_T,

$$I_T = 2C_S L_n$$

where C_S = Calculated section cathode current/unit length

L = Length

n = Number of cathode stripes

The computer program was, therefore, written based on the geometry shown with the assumption that the next set of points on the other side of the boundary line of the section would be a mirror image of the first set of points within the section. The program was written in a flexible manner so that the size of the electrodes and the spacing between them could be inputted. The program was originally written for a triode, but was later modified to cover the tetrode (in fact the program was general enough to cover a pentode form with a repeller beyond the anode in order to simulate devices constructed by Electron Emission Systems). The general approach to the solution will be discussed briefly. A copy of the final program (COPOIP) Coplanar Poisson Solution Pentode, is included as Appendix A. The program is written in ALGOL and is designed to operate on a remote terminal of the Burroughs 5500 computer. Many of the results presented later, however, were obtained with the original triode program.

The program calculates the potential at all points in space as a function of the potential of the various electrodes on the plane. There are a number of artifices built into the program as a result of the way it has been written. They are as follows:

1. The thickness of the metal electrodes is automatically one matrix unit high, (the dimension of the matrix is an inputted variable) except for the raised grid design which will be discussed below.

2. The substrate in-between electrodes is assumed to be at cathode potential. This is not necessarily an accurate assumption, but greatly simplifies the calculation process. The program could be refined to take into

account the dielectric constant of the substrate and the actual potential of various parts of the surface.

3. Although the boundaries at the edges of the section can change continuously, the last point in each row of the matrix above the coplanar plane is fixed at zero (cathode potential). The calculations made, thus, are for a coplanar device with a grounded metal plane above it. Studies were made of the effect of varying the number of points from the operating plane to the grounded plane, and it was found that there was a large effect which did not tend to approach saturation until 21 points above the operating plane were included. At this point the program running time exceeded the time permitted for a single run (ten minutes at the time these tests were run). The computer was later constrained to 3 minutes on direct processing and ten minutes on "scheduled" processing. The actual asymptotic value was never found except by the extrapolation shown in Figure 3. The program was revised to permit the last point in each row to be adjusted to a value other than zero and while this gave higher values of current and lower values of μ, the value which would be calculated if the last point was an infinite number of points away from the plane was not reached, i.e., the current still varied as a function of the number of points used in the row. It was, therefore, decided to let the last point remain at zero potential. This gave a result that was valid for a metallic plane above the active plane, which could be a valid configuration in a final package. For configurations without a metallic plane, a correction curve could be used based on the data collected and shown in Figure 3. Using this curve, a convenient number of points, but not less than 12, could be used for the actual computer computation, and the infinite number of points value could be obtained by multiplying the value obtained by the ratio of the infinite value (i.e., 15) to the value found on this curve for the number of row matrix points used.

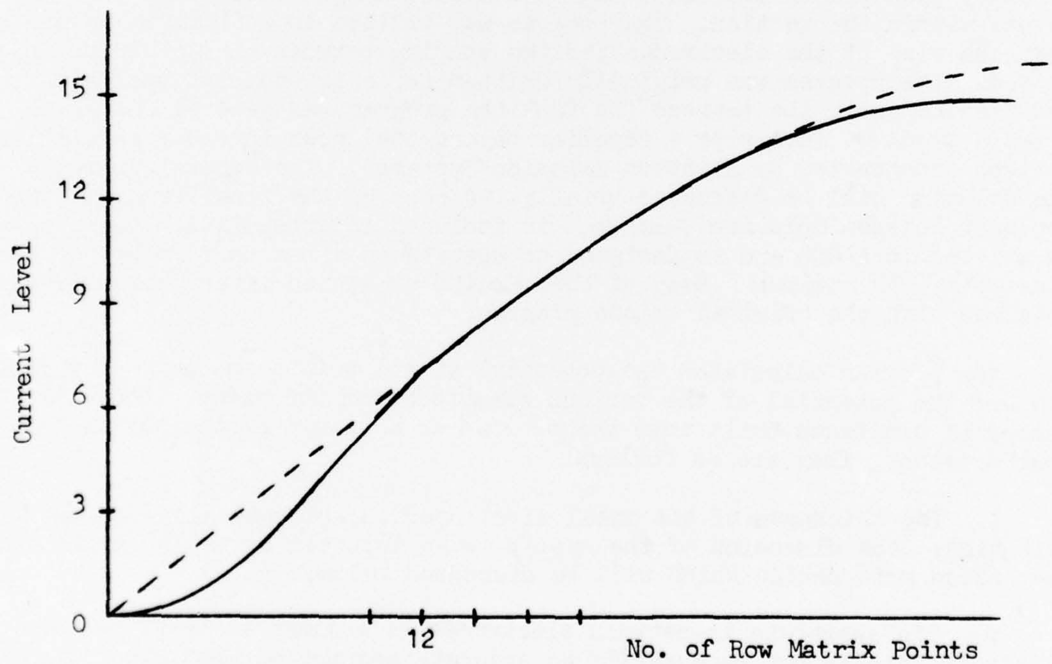


Figure 3. Calculated Current as a Function of Matrix Points Above Plane.

The first iteration of the program calculates the matrix point potential in the absence of space charge (the Laplace solution). A calculation is then made of the current drawn from the cathode based on the assumption that equation (1) is obeyed for the planar diode consisting of the cathode and the first set of matrix points immediately facing the cathode.

Instead of calculating exact electron trajectories from a reasonably large number of points at the cathode, the program then proceeds, by assuming that the charge at any given matrix point will flow to adjacent matrix points at a higher potential than the matrix point being checked, with the charge dividing in accordance with the ratio of the field between two points and the sum of the fields between the point from which current is flowing and all surrounding points. The points shown in Figure 4, are used to derive a typical charge flow expression.

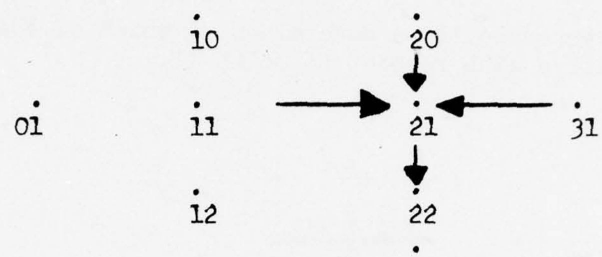


Figure 4. Arbitrary Set of Matrix Points Used for Demonstration of Charge Flow Conditions.

Assuming that the voltage at point 21 is greater than point 11 then

$$Q_{21(11)} \quad \alpha \quad Q_{11} \frac{E_{21-11}}{E_{21-11} + E_{12-11} + E_{10-11}}$$

or

$$Q_{21(11)} \quad \alpha \quad Q_{11} \frac{E_{21-11}}{E_{21-11} + E_{12-11} + E_{01-11}}$$

or

$$Q_{21(11)} \quad \alpha \quad Q_{11} \frac{E_{21-11}}{E_{21-11} + E_{10-11} + E_{01-11}} \qquad (8)$$

or

$$Q_{21(11)} \quad \alpha \quad Q_{11} \frac{E_{21-11}}{E_{21-11} + E_{12-11}}$$

or

$$Q_{21(11)} \quad \alpha \quad Q_{11} \frac{E_{21-11}}{E_{21-11} + E_{10-11}}$$

or

$$Q_{21(11)} \quad \alpha \quad Q_{11} \frac{E_{21-11}}{E_{21-11} + E_{01-11}}$$

where $Q_{21(11)}$ = Amount of charge flowing to point 21 from point 11

E_{21-11} = Electric Field between points 21 and 11

E_{12-11} = Electric Field between points 12 and 11

E_{10-11} = Electric Field between points 10 and 11 etc.

The equation selected would be determined by which of the neighbor points were positive with respect to point 11.

Since $E_{21-11} = \dfrac{V_{21} - V_{11}}{d}$

where V_{21} = the potential at point 21

V_{11} = the potential at point 11

d = the matrix spacing (constant for all points)

The first relationship shown in Equation (8) would reduce to

$$Q_{21(11)} \quad \alpha \quad Q_{11} \frac{(V_{21} - V_{11})}{(V_{21} - V_{11}) + (V_{12} - V_{11}) + (V_{10} - V_{11})}$$

or

$$Q_{21(11)} \quad \alpha \quad Q_{11} \frac{(V_{21} - V_{11})}{V_{21} + V_{12} + V_{10} - 3V_{11}}$$

To eliminate the proportional sign, we must take into account the current continuity equation, i. e., the current leaving point 11 must equal the current arriving at point 21 from 11, or, if the current from 11 were all going to 21 then,

$$I_{11 \rightarrow 21} = I_{21 \leftarrow 11}$$

$$P_{11}\upsilon_{11} = P_{21(11)}\upsilon_{21}$$

$$Q_{11}\upsilon_{11} = Q_{21(11)}\upsilon_{21}$$

$$Q_{21(11)} = Q_{11}\frac{\upsilon_{11}}{\upsilon_{21}} = Q_{11}\sqrt{\frac{V_{11}}{V_{21}}} \tag{9}$$

where $I_{11 \rightarrow 21}$ = Current flowing from 11 to 21

$J_{21 \leftarrow 11}$ = Current arriving at 21 from 11

υ_{11} = Velocity of electrons at 11

υ_{22} = Velocity of electrons at 21

This assumes that electrons left the cathode at zero velocity. For the case where the current divides between two or more points,

$$Q_{21(11)} = Q_{11}\sqrt{\frac{V_{11}}{V_{21}}}\left(\frac{(V_{21} - V_{11})}{V_{21} + V_{12} + V_{10} - 3V_{11}}\right) \tag{10}$$

and similarly for all the other possibilities of Equation (8).

Finally, the total charge at Q_{21} for the flow conditions shown by the arrows is calculated for Equation (11),

$$Q_{21} = Q_{21(11)} + Q_{21(20)} + Q_{21(31)} - Q_{21(22)} \tag{11}$$

After the total equilibrium charge flowing through each point is found, a second potential calculation iteration can be performed with the potential at each point being calculated based on the effects of nearest neighbor potentials and space charge reduction of potential, as has been derived by a number of authors.[5]

$$V_{21} = \frac{V_{11} + V_{20} + V_{31} + V_{22} + \frac{P_{21}h^2}{E}}{4}$$

$$V_{21} = \frac{V_{11} + V_{20} + V_{31} + V_{22} - \frac{Q_{21}}{QF}}{4} \tag{12}$$

where E = Permittivity of vacuum

P_{21} = Charge density at 21

h = Distance increment between points

5. Ramo, Whinnery and Van Duzer, Fields and Waves in Communications Electronics, pp. 165, John Wiley & Sons, (1967).

Using Equation (12), corrected potentials are calculated for all of the matrix points. A record is also kept of the amount of change in potential so that the average change in potential can be calculated. The process of recalculating the potentials is continued until the average change is less than a level fixed in the program. In the process of these calculations, charges arriving at positive electrodes are automatically obtained and are translated into both current density and total current per matrix length per electrode. Printout of current density, electrode current and, if desired, potential and charge at each matrix point are obtained after the iteration process has been terminated.

Using the program, values of plate current vs. grid voltage could be calculated. By increasing the grid bias gradually, the cut-off of the device could be calculated by finding the value at which the plate current is reduced to zero, and since at I = 0 from Equation (1) and (2), we obtain

$$I_b = GE_d^{3/2} = G\,(\mu E_g + E_b)^{3/2} = 0$$

$$\mu E_g + E_b = 0 \qquad (13)$$

$$\mu = -\frac{E_b}{E_g} = \left|\frac{E_b}{E_g}\right| .$$

The transconductance was estimated by calculating the plate current change due to a small change in grid voltage at constant plate voltage.

$$G_m = \frac{I_{b2} - I_{b1}}{E_{g2} - E_{g1}} = \frac{\Delta I_b}{\Delta E_g} \approx \left(\frac{dI_b}{dE_g}\right)_{E_b} \qquad (14)$$

Examination of the program as written indicated that the relationship between current or current density and anode voltage obeys the three-halves power law. The currents therefore, are, calculated using a value of 1.0 volt for the anode voltage and zero volts for the cathode voltage. All other electrode potentials are scaled relative to 1.0 volt. The coplanar tube thus obeys the Child-Langmuir equation as described by Equation (1). To obtain the actual current at a specific voltage, we use

$$I_{act} = I_{calc}\left(\frac{V}{1\text{ volt}}\right)^{3/2} \qquad (15)$$

The only scaling law to be derived from the program, then, was the dependence of the perveance constant, G, on the geometry and spacing. Data was collected using the program written for the triode but with no grid present. The data is tabulated in Table I. An examination of the data indicated that the current passed through a maximum value as the cathode size was increased at large anode to cathode spacings. The data for this condition is not included in the table and was not used in the curve fitting program described below since it would not meet the assumptions inherent in the equation for which a fit was sought (i.e., a linear log relationship). The conclusions drawn below from the derived equation apply only for relatively small cathode sizes.

TABLE 1. VARIATION OF ANODE CURRENT AS A FUNCTION OF CATHODE AND ANODE SIZE AND CATHODE TO ANODE SPACING

Cathode Size	4	4	4	4	4	3	2	1
Anode Size	8	7	6	5	4	4	4	4
Anode Current / *d_{pk}	$I \times 10^9$	$I \times 10^9$	$I \times 10^9$	$I \times 10^9$	$I \times 10^9$	$I \times 10^9$	$I \times 10^9$	$I \times 10^9$
8	-	-	-	-	8.8	9.6	10.4	10.1
7	-	-	-	-	14.4	15.7	17.0	16.7
6	-	-	-	-	23.7	25.9	28.2	27.9
5	42.2	42.0	41.7	40.9	39.5	43.2	47.3	47.6
4	70.9	70.6	70.0	68.8	66.5	72.9	80.5	82.9
3	120.8	120.3	119.4	117.4	113.6	124.9	139.4	147.7
2	208.5	207.7	206.2	203.0	196.7	216.7	244.9	269.0
1	360.7	359.3	356.8	351.7	340.4	375.9	430.9	492.7

* d_{pk} = number of units spacing between plate and cathode.

Cathode and anode size are normalized.

The data was fitted to the following equation with anode voltage = 1 volt.

$$I = (kA_c^{b_1} A_p^{b_2} d_{pk}^{b_3}).$$

or $\ln I = \ln k + b_1 \ln A_k + b_2 \ln A_p + b_3 \ln d_{pk}$

where A_k = Cathode area in (matrix units)^2

A_p = Anode area in (matrix units)^2

k, b_1, b_2, b_3 = Coefficients to be determined by statistical analysis.

Using the techniques of method of moments[6] for the analysis, the following results are obtained.

$$I = \frac{4.25\, A_p^{.03}\, A_k^{.009}}{d_{pk}^{1.32}} \times 10^{-7} V^{3/2} \frac{\text{Amps}}{\text{matrix unit of length}} \qquad (16)$$

This form of the equation is not of general use. The correlation was, therefore, repeated for an actual set of areas and spacings with the following results

$$I = \frac{4.89\, A_p^{.002}\, A_k^{.0005}}{d_{pk}^{1.33}} \times 10^{-8} V^{3/2} \frac{\text{Amps}}{\text{Cm of length}} \qquad (17)$$

where Ap and A_k are now expressed in cm^2 and d_{pk} in cm.

The perveance is, thus, not a very strong function of the electrode areas and varies inversely with the 4/3 power of the plate to cathode spacing instead of the strong dependence on cathode area and the inverse variation with the square of the spacing for a conventional bi-planar diode. Basically the cathode utilization is not uniform in the coplanar case, and the actual utilization varies with spacing and size to compensate somewhat for the difference in spacing. As a consequence of this non-uniform utilization, one must be sure that the maximum current density is not exceeded along the leading edge of the cathode.

Insufficient data was collected during the course of the program to permit a similar calculation of perveance of a triode to be made. One cannot automatically substitute an equivalent diode voltage for the triode case as

6. M. Ezekiel, Methods of Correlation Analysis, pp. 190-201, John Wiley & Sons, (1948).

is done in the multiplanar cathode. The general behavior, however, should be similar. The primary complicating factor will be that the size of the grid electrode will affect the plate to cathode spacing as well, which makes the relationship a more complicated one than in the conventional device.

While computer calculations of triode characteristics were being made, the undersigned conceived the idea that it would be possible to construct a "coplanar" device with the grid on the same substrate but extending to a higher level than either the cathode or the anode. The program was written to permit elevating the grid any number of matrix element spacings above the other electrodes. Computer runs (using the triode program) were performed for the case where the grid lies one matrix element higher than the cathode and anode. This results in higher values of amplification factor, μ, and transconductance to plate current ratio, R. This design was chosen for the power amplifier since it tended to minimize the length of electrodes required for a given power output, as will be shown later.

RESISTIVITY OF SUBSTRATES AS A FUNCTION OF TEMPERATURE

In order for the coplanar concept to be successful, it is necessary that the substrate used have a high resistivity at the operating temperature in order that bias potentials can be maintained on control electrodes. Kohl[7] reviews the data originally published by Campbell,[8] for a number of ceramics. Based on this data, only alumina and beryllia, of the ceramics conventionally used as electron tube envelopes, have a high enough resistivity at 700°C to be considered. The data shown for beryllia and magnesia is conflicting; the two sets of data differ by almost six orders of magnitude. Neither reference explains the differences, nor do they cite the original experiments so that the reader can determine the source of this difference.

Based on the data, however, it was decided to build the first coplanar devices using alumina substrates. In fact, to minimize the connection problem, a seven pin miniature ceramic stem was used with the pins cut-off flush with the normal inside surface. The metal electrodes of the coplanar triode were designed to use five of the seven pins as contacts and were deposited accordingly. The first devices processed were completed tubes. When processing was completed, the leakage between electrodes was far greater than would be predicted by the published high temperature ceramic data. An investigation was, therefore, made of the resistivity of the seven pin alumina miniature stem and other materials using only the grid-anode pattern of the triode. This would enable us to determine whether the resistivity difference was due to the ceramic itself being different from the sample covered in the literature, or whether the presence of active materials such as barium caused the low resistivity. Tests on the seven pin miniature stems (whose alumina purity could not be determined from our records), a number of alumina (99.5 percent purity)and of beryllia(99.5 percent purity),were performed.The results are shown in Figure 5. Tests performed on the alumina included a comparison of the use of chemical clearing of the substrates only prior to evaporation of the metal pattern, and the use of a 1000°C air firing after chemical

7. W. E. Kohl, Materials and Techniques for Electron Tubes, Reinhold, (1960).

8. I. E. Campbell, High Temperature Technology, John Wiley & Sons (1956).

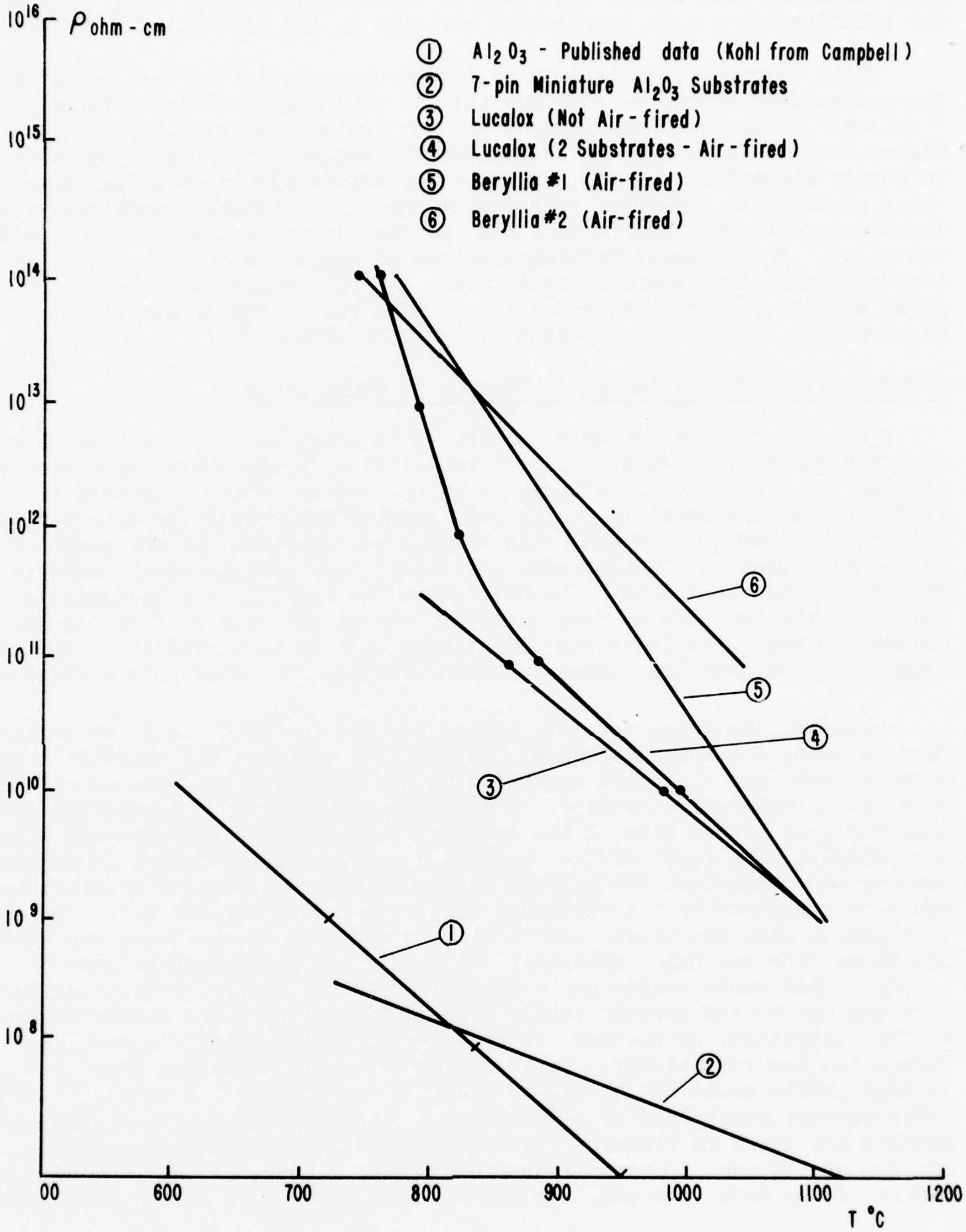


Figure 5. Resistivity of Substrates at High Temperatures

cleaning but before pattern evaporation. The following conclusions can be drawn from this series of experiments.

Limitations Due to Thermionic Emission Capabilities of the Photolithographic Cathode

1. The resistivity of the seven pin miniature stem at the temperature of interest was lower than the published values for alumina, but still high enough to have been useful, if other factors were not present.

2. The resistivity of high purity alumina is much higher than that shown in the literature and should be highly adequate for the coplanar substrate unless other factors, such as the presence of barium, come into play.

3. Air firing of the alumina results in a substantial improvement in resistivity particularly at lower temperature levels. The air firing cleaning step was adopted as a standard procedure on all substrates used thereafter.

4. The air-fired, high-purity beryllia data was within an order of magnitude of the higher level given by Campbell. The difference could be accounted for either by the approximation used to calculate resistivity for the highly non-standard configuration used for these measurements, or could represent sample variation (which was present in the samples tested). Unfortunately, time did not permit a comparison of non-air-fired material, which might have accounted for the wide range of data shown by Campbell. High purity beryllia, however, could serve as a useful substrate for coplanar devices barring interactions caused by the presence of active material.

The electron emitting material used in coplanar devices is one of the conventional mixes of triple (barium, strontium and calcium) carbonates used in thermionic devices. The primary difference is that in order to delineate the pattern of the cathode material, so that it is superimposed completely on a deposited metallic cathode substrate, the carbonates are placed in a binder of a photographically developable resist (KTFR) instead of the conventional amyl-acetate and nitro-cellulose binder usually used. In order to determine the effects of this binder and to verify the claims of low temperature emission capabilities,[1] a separate program was established to evaluate the cathode. The test program is covered in separate reports,[9,10] and will only be briefly summarized here. The major results have been:

(a) Tests on conventional nickel sleeve cathodes have shown useful emission capabilities in the 650°C region. While this is not quite as low as the 600°C quoted by Dore, Geppert, etc.[1] it still represents an improvement in low temperature operation over either standard oxide cathodes or the

1. B. Dore, D. Geppert, & R. Mueller, "Low Temperature Thermionic Emitter," NASA Contract No. NAS 12-607.

9. B. Smith, "Thermionic Emission from Oxide Coated Cathodes at Low Temperatures," USAECOM Technical Report, ECOM-3585, June 1972.

10. B. Smith. "Low Temperature Thermionic Cathode," Proc. IEEE-AGED Conf. on Electron Device Techniques, May 1973.

barium tungstate cathode. Life of over 5600 hours at a temperature of 650°C and an emission current density of 300 mA/cm^2 has been demonstrated.

(b) Delineated cathodes on coplanar substrates have been operated for over 5000 hours at a peak current density of 100 mA/cm^2. This operation has been obtained using a curve tracer as the operational test set. The substrate was fabricated by an outside source, but mounted and activated at this Laboratory. Substrates manufactured and activated by the existing source have been life tested under dc conditions at low current densities (tests were designed to detect charging effects in the substrate, which have not been observed) with no change in tube characteristics after 5000 hours of operation.

(c) Delineated cathodes fabricated at this laboratory have failed after operation up to 1000 hours. The cause of failure is not completely identified, but, in view of the other results obtained, it is believed to be due to poor adherence of the oxide cathode coating to the evaporated cathode electrode.

Limitations Due to Unwanted Thermionic Emission

Since all of the electrodes of a coplanar device are heated to the temperature on the substrate, a major question to be resolved is whether primary emission from control electrodes or from the anode will be experienced. To minimize this problem, we chose to use titanium as the electrode material for both the grid and the anode, while tungsten was used as the cathode material. Titanium has the property of reducing barium oxide that comes in contact with it, absorbing the oxygen, and releasing the barium which may then be deposited on a cooler structure. It should, therefore, be free from primary emission problems in the temperature range of the device. Although many problems with reverse grid current were encountered, these were all identified as leakage currents. No evidence of unwanted emission was noted. On the substrates and devices purchased for evaluation, which used a proprietary alloy for the control and collector electrodes, there also were no signs of unwanted emission. It is, therefore, considered highly feasible to fabricate coplanar devices and only obtain emission from the desired surfaces.

Triode Design and Performance

To verify the design of the coplanar triode, two types of tubes were planned. One was a simple triode whose basic purpose was to verify the computer calculations; the second was a power amplifier tetrode design to meet a specific objective. All of the substrate problems discussed above were noted with the triode design. These problems delayed the final stages of the program to the point that they were never executed. The tetrode design, however, did include one novel feature which may find application elsewhere, and for this reason, it will be discussed later.

The triode design chosen to check the computer program consisted of a device with four cathode stripes, a meanderline grid with eight full legs, and five anode stripes. The tube thus had eight of the unit sections shown in Figure 2. Although the original triodes were built with a diode-coupled input and a double diode (i.e., resistive) plate coupling, these features

were eliminated in later versions of the tube. The final tube was, therefore, of the type shown in Figure 6.

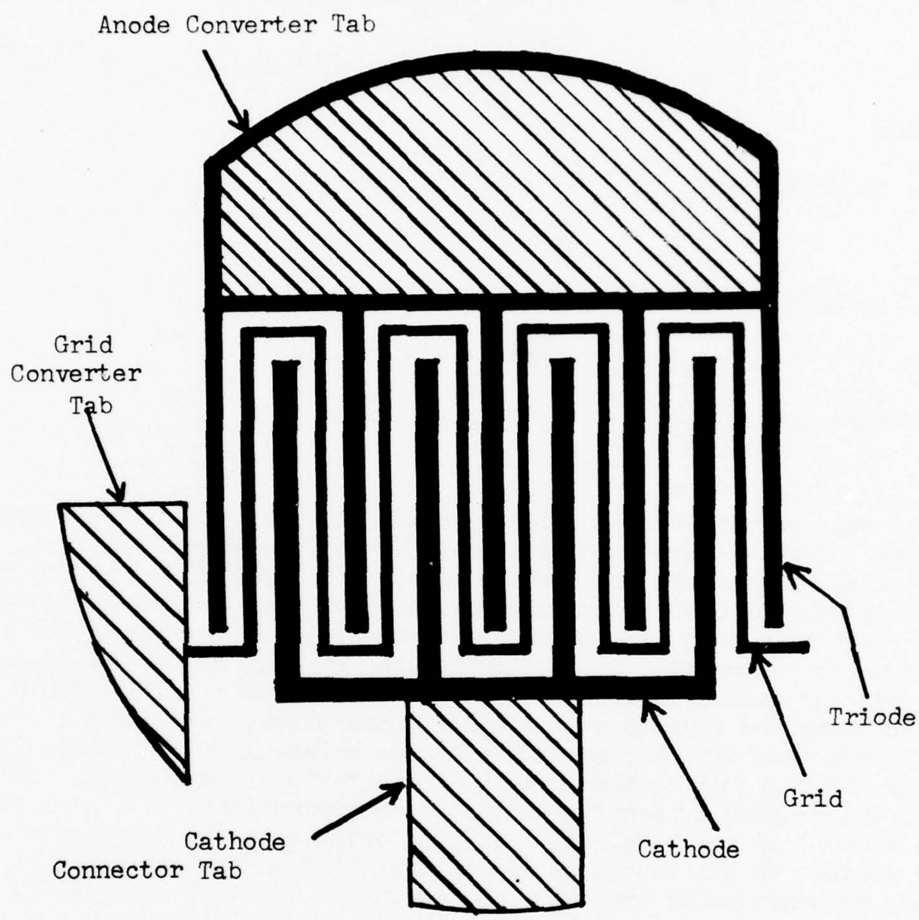


Figure 6. Triode Configuration

The dimensions of the various electrodes were designed to have the dimensions shown in Table 2. Also shown in the table are the number of matrix units corresponding to the dimensions for comparison with the computer calculations.

TABLE 2. TRIODE DESIGN FABRICATED

Element	Length (in.)	Matrix Units($\lambda = 6.35 \times 10^{-5}$m)
½ Cathode	0.005	2
Cathode to Grid Space	0.015	6
Grid	0.005	2
Grid to Cathode Space	0.010	4
½ Anode	0.005	2
Total Useful Length	1.875	-

The test results for the first successfully activated triode are shown in Figure 7 for the zero grid bias and -5 volt grid bias conditions. Also shown in this figure are the plate current values calculated by the COPOIT (triode version of COPOIP) program for the zero bias condition and two points for the -5 volt bias condition. The agreement for zero bias is considered to be reasonable, overestimating the current at the low plate voltage levels and underestimating it at the higher levels. The failure to check the computed results at the higher bias values is apparently due to the built-in bias of the program due to the maintenance of a plane above the coplanar structure at cathode potential as previously discussed. This artifice of the program evidently results in an overestimate of the μ (amplification factor) of the tube, which in turn results in the computer program giving greater weight to values of negative bias on the device than is actually encountered in the tube (the computed μ is approximately 5.8 for the geometry constructed, while the measured μ determined from cut-off data is approximately 2).

The tube for which data is presented was constructed on a sapphire substrate. No tubes were successfully constructed on the Lucalox (high purity alumina) substrates because of excessive leakage from grid to anode. It is, therefore, evident that some leakage mechanism is present in polycrystalline alumina (at least of the two types tested) that makes the material unsuitable for coplanar tube construction. Successful devices were constructed on polycrystalline beryllia so that the failures in alumina cannot be attributed to the polycrystallinity itself, but perhaps to the materials present at the grain boundaries in alumina compared to beryllia. The scope of the program did not permit further study of this problem.

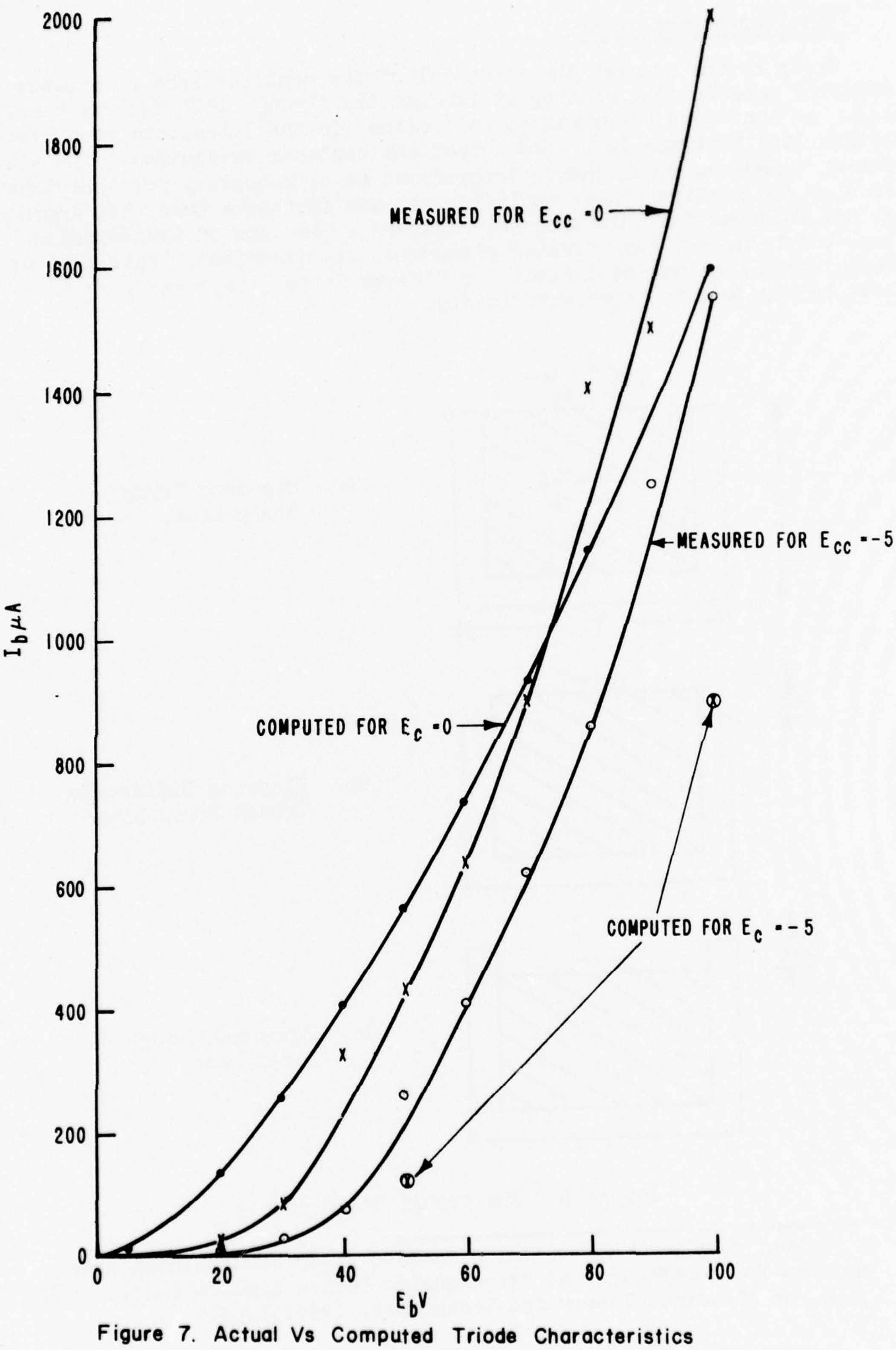
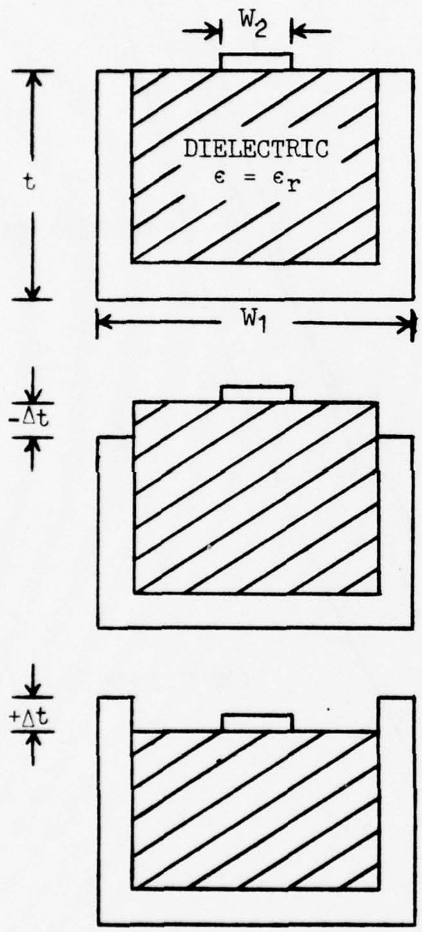


Figure 7. Actual Vs Computed Triode Characteristics

The Trough Strip Line

Early in the program the potential of the coplanar tube as a power amplifier capable of operating at frequencies through 1500 MHz was considered. At that time there was no indication in the literature of a transmission line that was later designated the coplanar waveguide.[11] In view of high impedance requirements later found to be necessary for individual lines of the coplanar power amplifier, it was fortunate that this approach was not followed for this program. Instead a new type of transmission line, which has not been treated elsewhere, was conceived. This type of transmission line was designated the "Trough Strip Line," and is shown in Figure 8 in three embodiments.



8a. Standard Trough Strip Line.

8b. Elevated Dielectric Trough Strip Line.

8c. Elevated Trough Strip Line.

Figure 8. The Trough Strip Line

11. C. P. Wen, "Coplanar Waveguide: A Surface Strip Transmission Line Suitable for Non-Reciprocal Gyromagnetic Device Applications," IEEE Trans. on Microwave Theory and Techniques, (Dec. 1969).

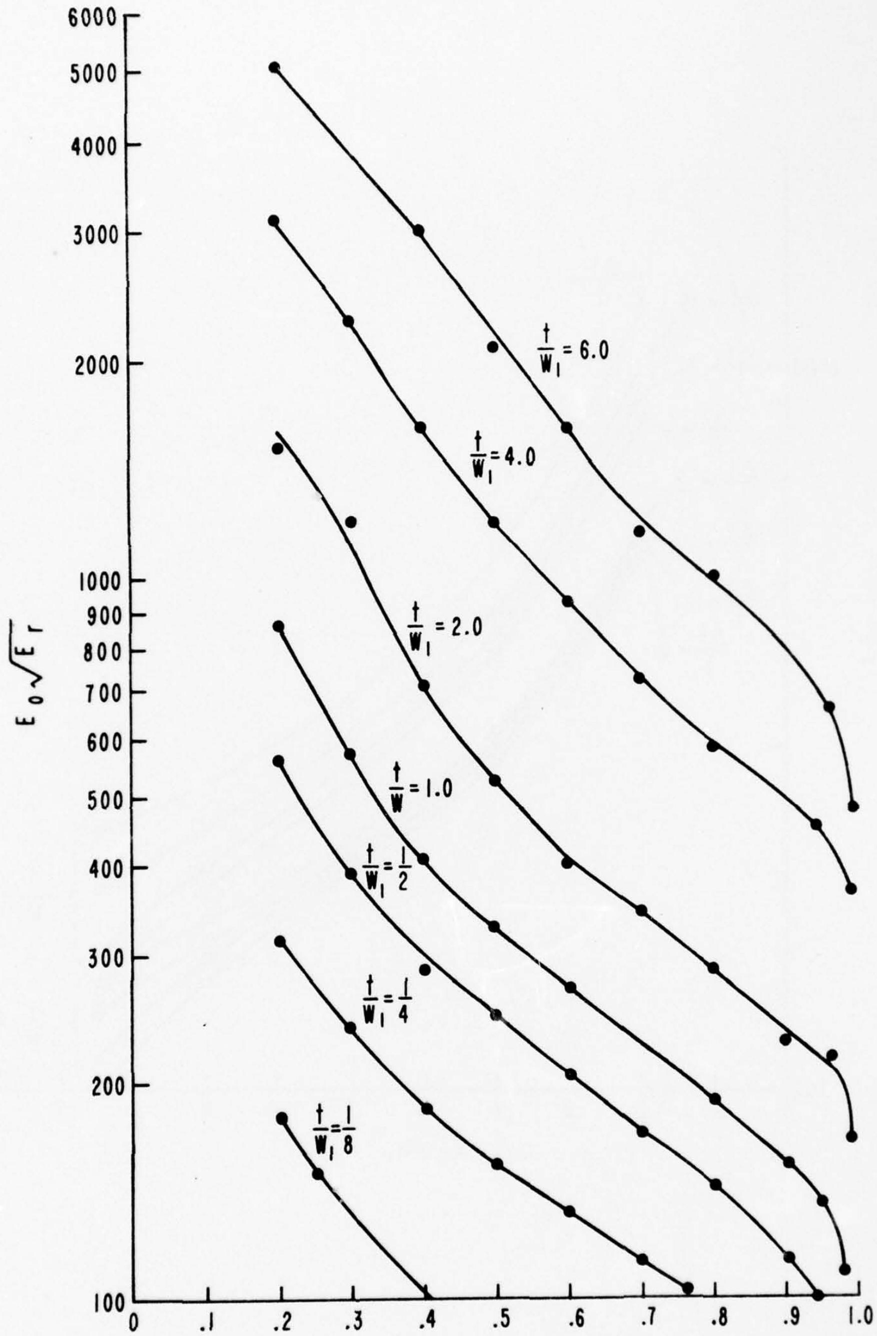


Figure 9. Trough Strip Line Impedance For Configuration Shown in Figure 8a.

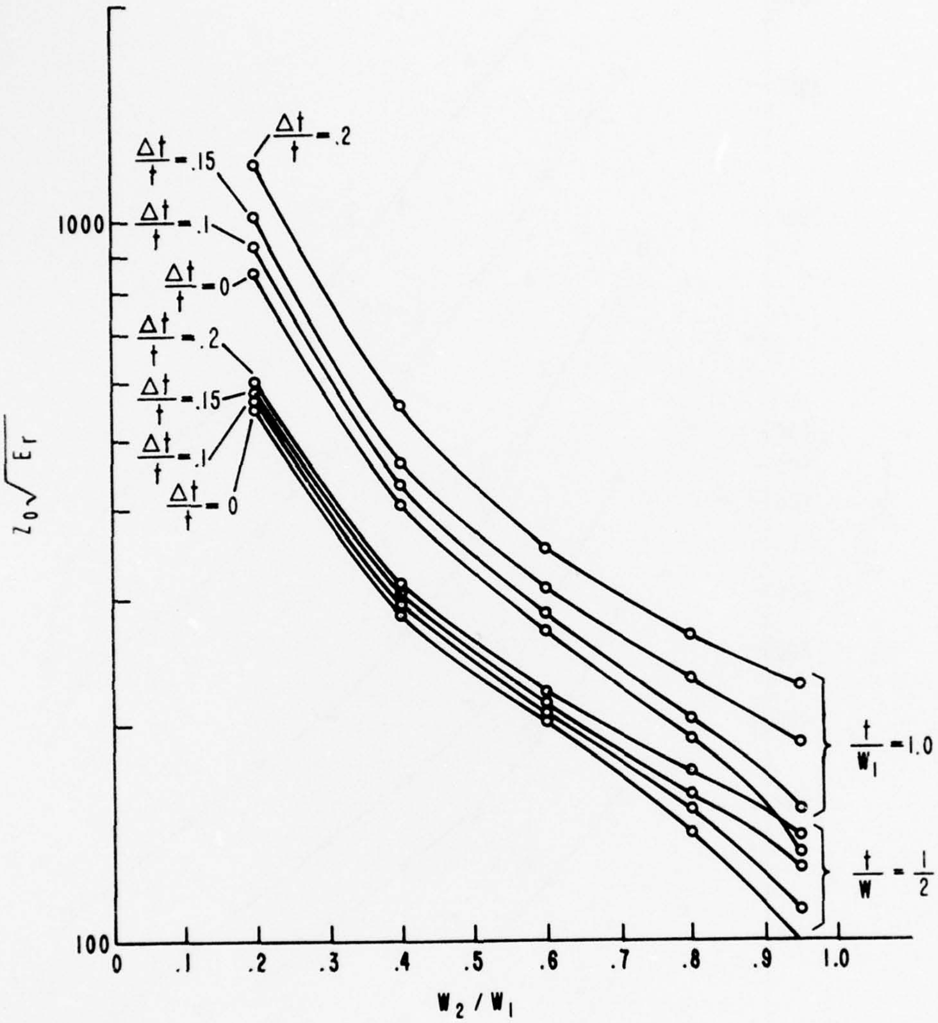


Figure 10. Trough Strip Line Impedance for Variation in Trough Sides as Shown in Figure 8b.

The case shown in Figure 8a was first checked using a model and gave sufficient promise to warrant analysis. The geometries of Figure 8a and 8b were analyzed by Capt. T. Freeman with the results shown in Figure 9 and Figure 10. The third case has not been analyzed as a transmission line, but, if the center conductor strip is considered to be a cathode and the two edges of the trough are the grids, the case can be recognized as that of a coplanar device with an elevated grid. The need for an elevated grid will be discussed below.

Power Amplifier Tetrode Design

To meet the power amplifier requirements, it was proposed to build a tube using the trough strip line by combining a number of such lines to form a pair of unit tube sections as shown in Figure 11.

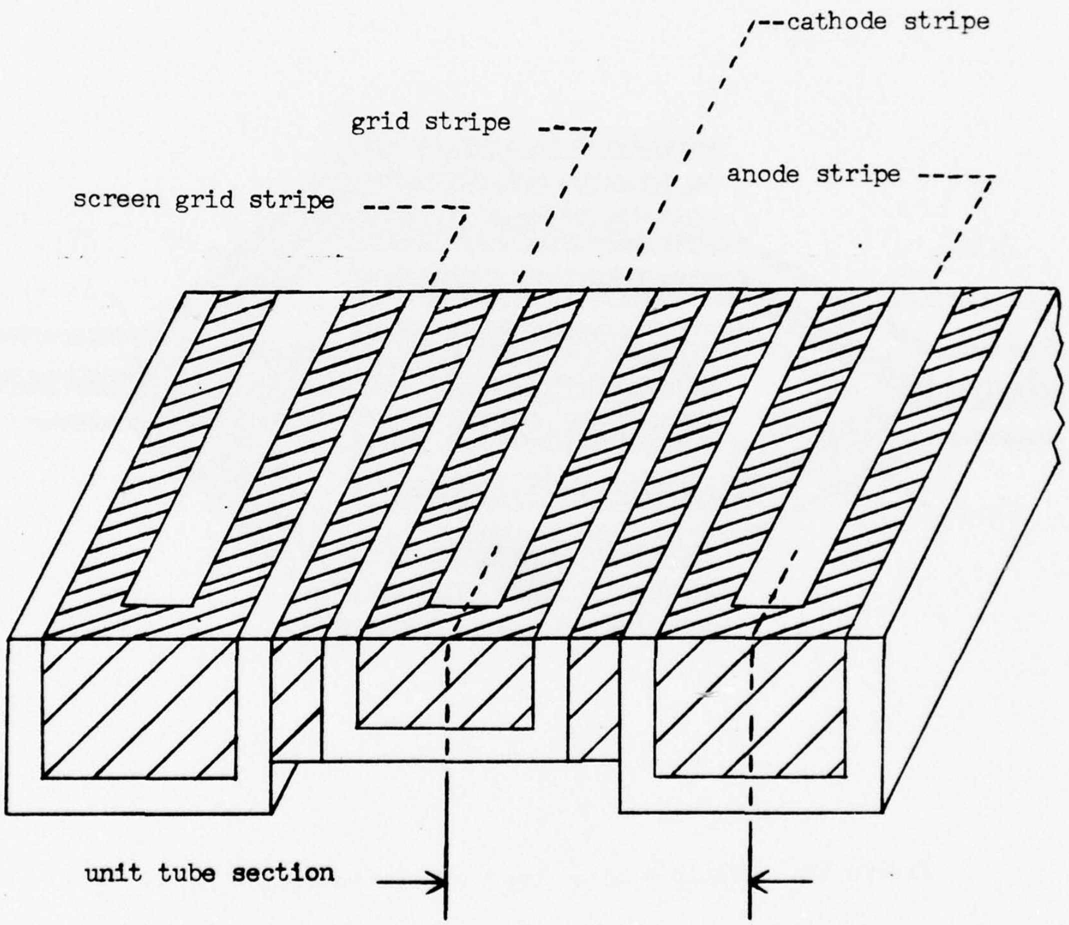


Figure 11. Trough Strip Line Tube Showing Two Tube Sections

If additional lines are needed in order to obtain the required device characteristics, they can be added by reiterating the unit tube section structure with the number of output lines always equal to one plus the number of input lines. When multiple lines are used, they would be diverged from the input and combined at the output, as shown in Figure 12, so that a single input and single output line could be used.

Figure 12 is illustrative in nature. If additional lines are required they could be achieved by combining groups of two inputs and three outputs or by other suitable combinations. It should be noted that the number of unit tube sections, where each unit section, as defined in Figure 1b, is equal to twice the number of input lines, i.e.

$$n_t = 2n_i . \tag{18}$$

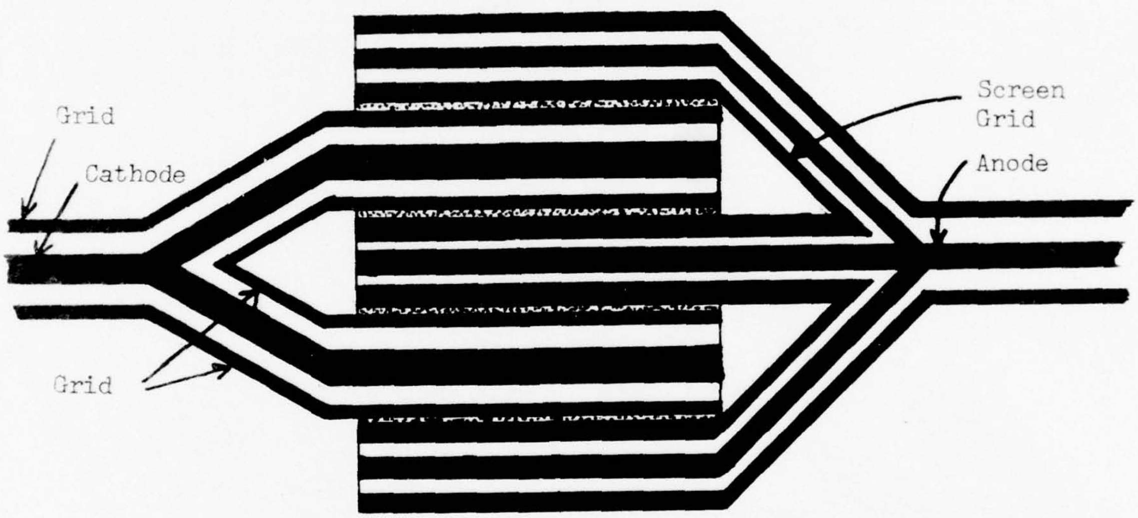


Figure 12. Multiple Line Input and Output ($n_o = n_i + 1$)

For the wide band trough strip line tube the following relationships apply

$$\text{Gain} = \frac{Zo}{Zi} \tag{19}$$

where Z_o = Output line impedance

Z_i = Input line impedance

and $$Z_i = \frac{1}{G_m} \tag{20}$$

where G_m = Transconductance

Equation (19) determines the input impedance required for a given output impedance (50 ohms is usually required) and a desired gain. Thus, for a gain of 25, an input impedance of 2 ohms would be required. For a gain of 20, the input impedance would be 2.5 ohms. The transconductance corresponding to the latter value of input impedance, based on Equation (20), is 400,000 μmhos.

A number of tube geometries were explored with the objective of meeting the required gain and transconductance, while minimizing a combination of active tube length and the number of unit sections. Calculations were made based on an objective of achieving 150 watts peak power at a duty of .06 with a gain of 20.

The design objectives and requirements for the tube are summarized in Table 3.

TABLE 3. TETRODE DESIGN OBJECTIVES

Factor	Symbol	Objective	Unit
Peak Power Output	Po	150	w
Duty	Du	0.06	
Output Impedance	Z_o	50	Ω
Gain	G	20	
Input Impedance	Z_i	2.5	Ω
Transconductance (1)	$G_m(1)$	400,000	μmhos
Transconductance (2)	$G_m(2)$	800,000	μmhos

In Table 3 the transconductance calculated from Equation (20) is listed as G_m (1). A value of twice this value must be achieved at the peak current level in order to meet the gain requirements at the fundamental frequency when operating as a single-ended Class B amplifier. This value is listed as G_m (2). The design computation and the equations used to make them are summarized in Table 4 for a tube operated as Class B amplifier.

All of the computations included in Table 4 are self-explanatory except for the calculation of minimum plate swing. This calculation uses the computer generated tube characteristics to determine the voltage at which the required peak current can be drawn under conditions which will result in the necessary transconductance. The equation is derived from the G_m to current ratio.

$$R = \frac{G_m}{i_{rf}} = \frac{G_m\ /\text{unit length}}{i} \qquad (21)$$

where i = current for unit length

$$\text{but}\quad \frac{G_m}{\text{unit length}} = \left(\frac{\partial i}{\partial e_c}\right)_{Eb} \approx \left(\frac{\Delta i}{\Delta e_c}\right)_{E_{min}} \qquad (22)$$

Let $\Delta e_c = f E_{min}$ where f is a standard small fraction of the constant plate voltage, E_{min}.

Substituting (22) in (21) we have

$$R = \frac{\Delta i/(f\ E_{min})}{i} \qquad (23)$$

Solving for E_{min}

$$E_{min} = \frac{\Delta i/i}{fR} \qquad (24)$$

The $\Delta i/i$ values can be taken directly from computer tabulations which are made at an arbitrary voltage of 1.0 volt. The value of E_{min} tabulated is based on a unit geometry as shown in Figure 13.

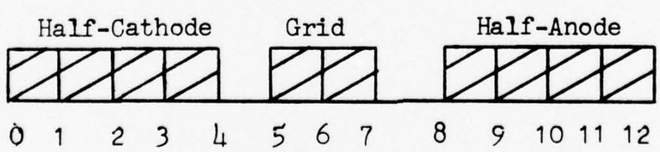


Figure 13. Unit Section Geometry. Each box is equal to 2.0×10^{-3} centimeters.

TABLE 4. TETRODE DESIGN COMPUTATIONS

Factor	Equation	Class B Amp.	Units
Peak Load Voltage	$e_L = (2P_oZ_o)^{1/2}$	122.5	v
Peak Load Current	$i_L = (2P_o/Z_o)^{1/2} = \frac{e_L}{Z_o}$	2.45	a
Peak Video Current	$i_b = i_L/1.11$	2.21	a
Average Current	$I_b = i_b \cdot Du$	0.132	a
RMS Current	$i_p = \quad i_bI_b$	0.541	a
Peak RF Current	$i_{rf} = \pi i_b$	6.93	a
Transconductance to Current Ratio	$R = C_m(2)/i_rf$	115.45	µmhos/µA
Min Plate Swing	$E_{min} = (\Delta i/i)(1/P_o)$ (Note 1)	42.15	v
Plate Voltage	$E_{bb} = e_L + E_{min}$	164.65	v
Power Input	$P_i = i_bE_{bb}$	363.9	w
Efficiency	$\eta = P_o/P_i$	41.2	%
Peak Current Density	$j_{rf}(max)$ (Note 2)	1.14	a/cm^2
RMS Current Density	$j_p(max)$ (Note 2)	0.089	a/cm^2

Note (1) - See Text for discussion and definitions of terms

Note (2) - Computed value for selected tube geometry

As can be seen, this is a triode geometry. At the time these computations were made, the computer program had not yet been modified to obtain tetrode calculations. Since the screen grid area used is small, it is assumed that most of the current will be collected at the anode. In the final design the screen-grid, insulator, and anode surface of the output trough transmission line will occupy the area of the triode anode.

We must now resolve the question of how shall the required active length be obtained. We can select either a long device with a minimum number of input and output lines or a short device with a large number of lines. The long tube would provide a satisfactory solution at low frequencies only, however, where the wavelength is much greater than the required active length of tube. To achieve our objective of operation through 1500 MHz would require a length which is small compared to 20 cm., i.e., on the order of $\lambda/10$ or 2 cm. Using this length with the geometry calculated would result in approximately 660 tube units or 330 input lines. While this may not be any more difficult to construct, as far as fabrication is concerned, than a larger design with say 50 input lines, it leads to problems with the selection of transmission line impedance since an individual output line would require an impedance of 50 X 330 = 165,000 ohms in order to achieve the desired 50 ohms output impedance.

The proposed coplanar design, with its evaporated electrode fabrication technique, lends itself readily to an alternative approach, i.e., the tapered-line impedance, distributed amplifier originally proposed by General Electric to improve distributed amplifier efficiency. Normally, the output line of a distributed amplifier must be terminated with its characteristic impedance at the input end of the line. This results in the loss of half of the output power. With a tapered line design having a high impedance at the input end tapering to the required impedance at the output end, the efficiency of the distributed amplifier can be doubled.[12] With the higher efficiency feasible, a distributed amplifier whose length is not limited to a fraction of a wavelength can be used. The required impedance taper can be achieved in a trough strip line by one of two methods or a combination of the two. The two basic methods are illustrated in Figure 14. Using this approach, we can select a convenient number of input lines or a maximum length we do not wish to exceed from a fabrication standpoint.

Before a final calculation is made, examination of the effect of elevating the grid will be made. To understand the effect, the contribution of the factor R, included in Table 4, must be determined.

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12. Final Report, Contract DA 36-039-SC-90743, "Distributed Amplifier Tube," General Electric Company.

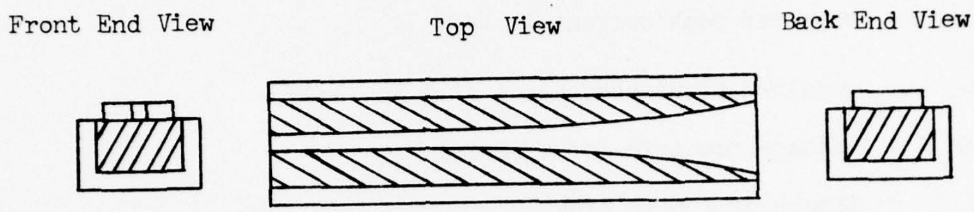

Figure 14a. Constant Thickness Variable Strip Width Impedance Tape.

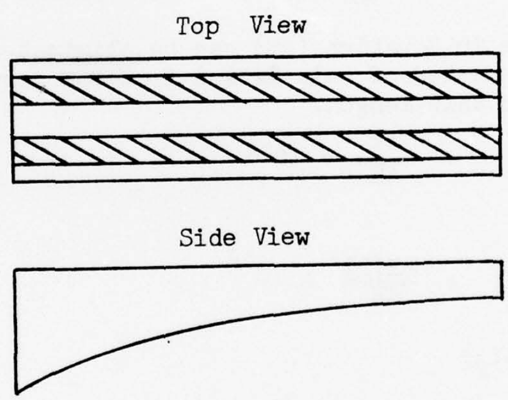

Figure 14b. Constant Strip Width Variable Thickness Tape.

To analyze this effect let

i = current per unit element at one volt in μA.

I_{50} = $(i/d)\ (50)^{3/2}$ $\mu A/cm$ = current per cm at 50 volts

d = dimension of unit element

$\hat{i}$ = required peak current in amps

$\hat{G}_m$ = required transconductance at $\hat{i}$ in mhos

Em = voltage required for $\hat{i}$ (E_{min})

L = length of active area

Iem = $(i/d)(Em)^{3/2}$ $\frac{\mu A}{cm}$ = current per cm at E_{min}

From the total peak current required and the peak current achievable at Emin for each centimeter length (Iem), we can determine the total length of cathode required, i. e.,

$$L = \frac{\hat{i}\ \text{amp}}{(Iem) \times 10^{-6}\ \text{amp/cm}} = \frac{\hat{i}}{(i/d)\ (Em)^{3/2} \times 10^{-6}}\ \text{cm} \tag{25}$$

The value of Em in Equation (25) can be eliminated by starting with the definition (and method of calculation from the computer data) of the transconductance per unit length.

$$\hat{G}_{m/L} = \frac{\Delta I}{\Delta E} \times 10^{-6}$$

or

$$\hat{G}_m = \frac{\Delta I(L)}{\Delta E} \times 10^{-6} \tag{26}$$

but $\Delta I = (I_1 - I_2)$

and $Iem \approx I_1$ when the change in E is small

$$\text{therefore } \hat{G}_m = \frac{\left(\frac{I_1 - I_2}{I_1}\right) Iem(L) \times 10^{+6}}{\Delta e\ \ Em} \tag{27}$$

where Δe is the standard change in voltage used compared to 1.0 volt in the computer calculations, and $\Delta E = \Delta e\ Em$.

But $Iem(L) = \hat{i} \times 10^{+6}$

or $$\hat{G}_m = \frac{\left(\frac{I_1 - I_2}{I_1}\right)\hat{i}}{\Delta e \, Em}$$

solving for Em and using the actual computer computations i_1, i_2 in the normalized current ratio

$$Em = \frac{\left(\frac{i_1 - i_2}{i_1}\right)\hat{i}}{\Delta e \, \hat{G}_m} \tag{28}$$

Substituting (28) in Equation (25) we have

$$L = \frac{\hat{i} \times 10^6 \, (\Delta e \, \hat{G}_m)^{3/2}}{\frac{i_1}{d}\left(\frac{i_1 - i_2}{i_1}(\hat{i})\right)^{3/2}} \tag{29}$$

Using the values $\hat{i} = 6.93$ amperes

$\hat{G}m = 0.8$ mhos

obtained from the Tetrode Design Plan and Δe .005 used in the computer run-offs Equation (29) becomes

$$L = \frac{96.1 \, d \, (i_1)^{\frac{1}{2}}}{(i_1 - i_2)^{3/2}} \tag{30}$$

Equation (30) is plotted in Figure (15) for a unit spacing of 1.015×10^{-3} cm for a number of values of $\Delta i = i_1 - i_2$. These are equivalent to specific values of transconductance at the voltage of 50 volts and the current drawn at that voltage. The values of transconductance per centimeter of length are also shown on the figure. Also shown are lines of constant R, where R is a figure of merit arbitrarily taken at the 50 volt condition and equal to

$$R = \frac{Gm_{50}}{I_{50}} \; \frac{\mu\text{mhos}}{\mu\text{A}}$$

One can see from the figure that geometries that lead to high values of R and high values of Gm per unit length result in smaller cathode areas to achieve a specific value of peak current and total transconductance at the peak current. The elevated grid coplanar tube gave just such a result.

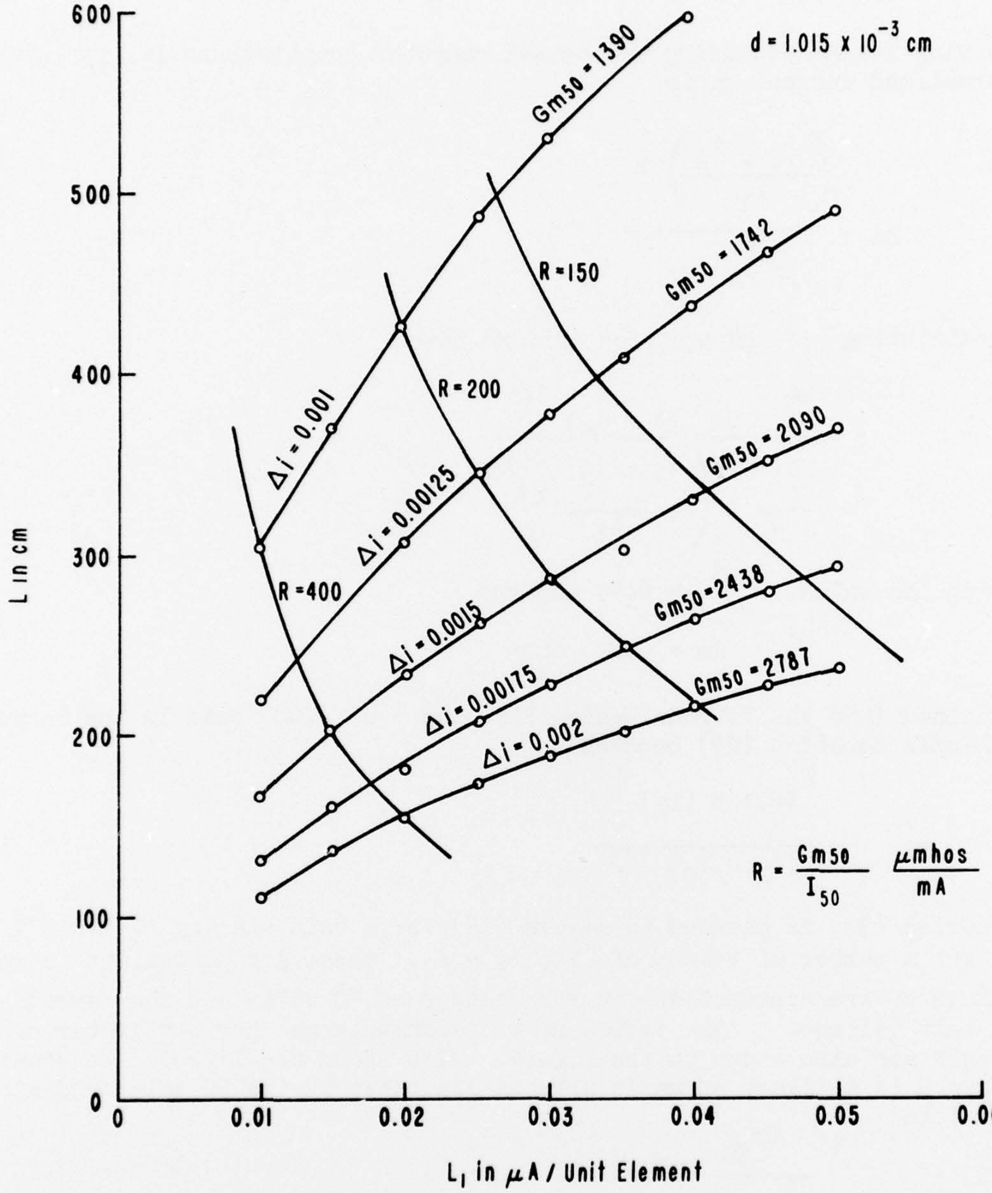

Figure 15. Required Cathode Length as a Function of Peak Current per Matrix Element.

For instance, at the unit element spacing used for Figure 15 using an unelevated grid with a Δ i of .000125, a Gm_{50} = 1742 μmhos/cm at an i_1 current of 7.1 X 10^{-8} amps per unit element was observed. The R value is far below the 150 μmhos/mA level included in the figure. The required length of active device to achieve the peak current and Gm required was 571 centimeters. In contrast to this, the same geometry, except that the grid was elevated off of the plane by one matrix unit, gave a Δ i of .000151, and a Gm_{50} = 2090 μmhos/cm at an i_1 current of 3.16 X 10^{-8} amps per unit element. The R value is approximately 200, and only 283 centimeters of active tube would be required to achieve this condition. For a unit spacing of 2.0 X 10^{-3} cm, one could design an elevated grid, trough strip line tube that was 13.9 cm long using 40 tube units (20 output lines) for a total active tube length of 558 cms at a characteristic impedance of the strip line of 1000 ohms, compared to a non-elevated grid tube 14.5 cm long using 80 tube units (40 output lines) for a total active length of 1124 cms at a characteristic impedance of the strip line of 2000 ohms. Based on these results, it was concluded that an elevated grid structure was desired for the trough strip line tube.

It should be pointed out that one cannot simply design for the highest possible R value, such as that given by the use of the elevated grid and the closer spacing or by going to still higher elevated grids. Aside from the obvious fact that a limit would be reached determined by the peak current density capability of the tube, it can be shown that the higher R is the higher the value of E_{min} and, therefore, the lower the efficiency of the device. This is not obvious from Equation 24, which seems to indicate the opposite result. From Figure 15, however, we can see that as R increased $\Delta i/i$ also increases and actually increases more rapidly than R as the grid is elevated above the plane.

Tubes of the tetrode design calculated above were never constructed because of lack of time, and because the methodology proposed to achieve the multiple strip line construction involved the use of polycrystalline alumina using evaporation and etch techniques developed by Heynick and coworkers.[13] Since the polycrystalline alumina failed to provide satisfactory triodes as discussed above, it was not considered worthwhile to attempt actual construction of the multiple element, trough strip line tetrode.

CONCLUSIONS AND RECOMMENDATIONS

The coplanar tube investigation was terminated based on the results described above for the following reasons: (1) While the reasons given for initiating the program, control of spacings and high temperature operation were verified, the tubes actually tested and the designs calculated from the computer programs did not show promise of exceedingly high performance devices. Indeed, the scaling law derived indicated that both tube element area and tube element spacing would play a lesser role than

13. Heynick and co-workers, Final Reports DA 28-043-AMC-01766, "Applied Research In Thin Film Field Emission Tubes," and DA 28-043-AMC-01261(E), "High Information Density Storage Surfaces," Stanford Research Institute.

that experienced in conventional tube design; (2) The wideband tetrode,[14] for which the coplanar tube was being investigated as a driver tube, was found to be less practical than projected at the start of this program.

Although the program was terminated with the conclusion that coplanar tubes would not play a major role in Army electronics, a number of portions of the program have yielded significant results with respect to the general body of scientific knowledge.

a. The results on the high temperature resistivity of the insulators investigated supplement the available information and point out the need for explicit inclusion in the literature of the conditions used for treating the samples used for test purposes. The experiments performed here should be repeated with more standardized electrode configurations to provide greater confidence in the absolute number calculated for resistivity. The role of active materials on polycrystalline alumina surfaces in determining resistivity should also be determined.

b. The trough strip line developed for this program should have application in conventional transmission line applications. In addition, the tapered impedance concept advanced here could be applied to a multiplanar distributed amplifier tube which would be of much simpler construction than distributed amplifier tubes previously developed.

c. The computer programs developed for the purpose of this effort may be of use in future coplanar programs, which are initiated to take advantage of the radiation immunity characteristics of the device, which were not particularly pertinent to existing Army requirements.

ACKNOWLEDGMENTS

The work of many people contributed to the study reported here. The assistance given by Capt. Thomas Freeman for his computer program contributions; Mr. Stanley DuBuske for activation and testing of triodes and pentodes; Mr. Bernard Smith for cathode preparation; Mr. Charles LoCascio and Mr. Martin Long for fixture designs; Mr. Edward Daly for metal evaporations; and Mrs. Mae Osborn and Mr. Albert Newman for tube assembly, is gratefully acknowledged.

It is also a pleasure to acknowledge the contributions of Mr. James Baxendale and Mr. George Taylor in the technical review, and revisions and corrections to the manuscript.

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14. Final Report DAAB07-69-C-0439, "Triode Tetrode Amplifier Development," Bendix Corporation.

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APPENDIX A

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↑COPOIP
BEGIN
$$A START1
"POIP1"
$$A START2
INTEGER  R,C,A,B,P,T,V,W,D,E,F,G,M,N,ITER,WR1,WR2,WR3,STT,DD,DE,WRØ:
REAL TU,S,H,DIF,IK,IS,IA,Q1,Q2,Q3,Q4,NUM,IG,DEN,AV,CT:
REAL ARRAY U[Ø:5Ø,-2:25],Q[Ø:5Ø,Ø:25],J[Ø:5Ø]:
LABEL L1,L5,L6,L7,L8,L9,L47,L48,L49,L5Ø,L51,L52,L53,L54,L55,L56,
                L57,L58,L59,L6Ø,L61,L62,L63:
FORMAT F1("TYPE M,N,A,BT,V,W,,<<<<]<<]<]=ZLVKR1,WR2,WR3,"):
 FORMAT F2 (5E14.6,/):
 FORMAT F3 (E14.6,/):
 FORMAT F4 ("IK IS"):
 FORMAT F5 ("IG IS"):
 FORMAT F6 ("IS IS "):
 FORMAT F7 ("IA IS"):
 FORMAT F8 ("JK IS"):
 FORMAT F9 ("JG IS"):
 FORMAT F1Ø ("JS IS"):
 FORMAT F11 ("JA IS"):
 FORMAT F12 ("ITERATIONS = ",I4,X5,"AVE. CHANGE=",E15.8):
 FORMAT F13(X2Ø,"VOLTAGE MATRIX"):
 FORMAT F14(X2Ø,"CHARGE MATRIX",/):
 FORMAT F15(X5,"EG1=",F4.2,"EG2=",F4.2,"ER=",F4.2):
 WRITE (TYPE,F1):
 READ (INPUT,M,N,A,B,P,T,V,W,D,DD,DE,E,F,G,H,WRØ,WR1,WR2,WR3):
 READ (DATA,FOR R:=Ø STEP 1 UNTIL M DO[FOR C:=Ø STEP 1 UNTIL N
        DO [U[R,C]]]):
 ITER:=Ø:
 L1:
 S:=Ø:
 ITER:=ITER+1:
 FOR R:=Ø STEP 1 UNTIL M DO
   BEGIN
   LABEL LA1,LA2,LA3:
   IF R LEQ A THEN GO TO LA1:
   IF R LEQ B THEN GO TO LA2:
   IF R LEQ P THEN GO TO LA1:
   IF R LEQ T THEN GO TO LA2:
   IF R LEQ V THEN GO TO LA1:
   IF R LEQ W THEN GO TO LA2:
   IF R LEQ D THEN GO TO LA1:
   IF R LEQ DD THEN GO TO LA2:
   IF R LEQ DE THEN GO TO LA1:
   IF R LEQ E THEN GO TO LA2:
   IF R LEQ F THEN GO TO LA1:
   IF R LEQ G THEN GO TO LA2:
   IF R LEQ M THEN GO TO LA1:
   LA1: FOR C:=2 STEP 1 UNTIL N-1 DO
      BEGIN
     TU:=(U[R-2\SIGN(R)+1,C]+U[R+2\SIGN(M-R)-1,C]+U[R,C-1]+U[R,C+1]
            -Q[R,C]/(8.8459-12))/4:
       DIF:=ABS(TU-U[R,C]):
       S:=S+DIF:
       U[R,C]:=TU:
       END:
```

```
    GO TO LA3;
    LA2: FOR C:=1 STEP 1 UNTIL N-1 DO
         BEGIN
         TU:=(U[R-1,C]+U[R+1,C]+U[R,C-1]+U[R,C+1]-Q[R,C]/(8.8460-12)
                   )/4;
         DIF:=ABS(TU-U[R,C]);
         S:=S+DIF;
         U[R,C]:=TU;
         END;
  LA3: END;
  FOR R:=B+1 STEP 1 UNTIL P DO
    BEGIN
    GO TO IF U[R,2] GTR U[R,1] THEN L5 ELSE L6;
    L5: Q[R,2]:=(3.930-12)\(U[R,2]-U[R,1])\H;
    J[R]:=Q[R,2]\(5.9305)\SQRT(U[R,2]-U[R,1])/H*3;
    GO TO L7;
    L6: Q[R,2]:=0;
    L7: END;
  FOR R:= B+1 STEP 1 UNTIL P-1 DO
    BEGIN
    IF U[R+1,2] GTR U[R,2] THEN GO TO L55;
    IF U[R-1,2] GTR U[R,2] THEN GO TO L59;
    L55:IF U[R-1,2] GTR U[R,2] THEN GO TO L58;
     Q[R+1,2]:=Q[R+1,2]+Q[R,2]\(U[R+1,2]-U[R,2])/(U[R+1,2]
                   + U[R,3]-2\U[R,2])\SQRT(U[R,2]/U[R+1,2]);
    GO TO L56;
    L59:Q[R-1,2]:=Q[R-1,2]+(Q[R,2]\(U[R-1,2]-U[R,2])/(U[R-1,2]
                   +U[R,3]-2\U[R,2]))\SQRT(U[R,2]/U[R-1,2]);
    GO TO L56;
    L58:Q[R+1,2]:=Q[R+1,2]+(Q[R,2]\(U[R+1,2]-U[R,2])/(U[R+1,2]
       + U[R-1,2]+ U[R,3]-3\U[R,2]))\SQRT(U[R,2]/U[R+1,2]);
    Q[R-1,2]:=Q[R-1,2]+(Q[R,2]\(U[R-1,2]-U[R,2])/(U[R+1,2]+U[
         R-1,2]+U[R,3]-3\U[R,2]))\SQRT(U[R,2]/U[R-1,2]);
     L56: END;
  FOR R:=0 STEP 1 UNTIL M DO
    BEGIN
    LABEL LB1,LB2,LB3,LB4,LB5,L10,L11,L12,L13,L14,L15,L16,L17,
             L18,L19,L20,L21,L22,L23,L24,L25,L26,L27,L28,L29,
              L30,L31,L32,L33,L34,L35,L36,L37,L38,L39,L40,L41,
                L42,L43,L44,L45,L46;
    U[R,-2]:=0;
    U[R,-1]:=0;
    IF R LEQ A THEN GO TO LB1;
    IF R GTR B THEN GO TO LB3;
    LB2: STT:=1; .
    GO TO LB4;
    LB3: IF R GTR P THEN GO TO LB5;
    STT:=3;
    GO TO LB4;
    LB5: IF R LEQ T THEN GO TO LB2;
    IF R LEQ V THEN GO TO LB1;
    IF R LEQ W THEN GO TO LB2;
    IF R LEQ D THEN GO TO LB1;
    IF R LEQ DD THEN GO TO LB2;
    IF R LEQ DE THEN GO TO LB1;
    IF R LEQ E THEN GO TO LB2;
    IF R LEQ F THEN GO TO LB1;
    IF R LEQ G THEN GO TO LB2;
     LB1: STT:=2;
     LB4: FOR C:=STT STEP 1 UNTIL N-1 DO
```

```
BEGIN
IF U[R,C] LEQ Ø THEN GO TO L1Ø:
IF U[R,C-1] LEQ Ø THEN GO TO L11:
IF U[R,C] LEQ U[R,C-1] THEN GO TO L11:
NUM:=Q[R,C-1]\(U[R,C]-U[R,C-1])\SQRT(U[R,C-1]/U[R,C]);
IF U[R-2\SIGN(R)+1,C-1] LEQ U[R,C-1] THEN GO TO L12:
IF U[R+2\SIGN(M-R)-1,C-1] LEQ U[R,C-1] THEN GO TO L13:
IF U[R,C-2] LEQ U[R,C-1] THEN GO TO L14:
DEN:=(U[R,C]+U[R-2\SIGN(R)+1,C-1]+U[R+2\SIGN(M-R)
        -1,C-1]+U[R,C-2]-4\U[R,C-1]);
IF DEN EQL Ø THEN GO TO L11:
Q1:= NUM/DEN:
GO TO L15:
L12: IF U[R+2\SIGN(M-R)-1,C-1] LEQ U[R,C-1] THEN GO TO L16
IF U[R,C-2] LEQ U[R,C-1] THEN GO TO L17:
DEN:=(U[R,C]+U[R+2\SIGN (M-R)-1,C-1]+U[R,C-2]
        -3\U[R,C]):
IF DEN EQL Ø THEN GO TO L11:
Q1:= NUM/DEN;
GO TO L15:
L13: IF U[R,C-2] LEQ U[R,C-1] THEN GO TO L18:
DEN:=(U[R,C]+U[R-2\SIGN (R) +1,C-1]+U[R,C-2]
        -3\U[R,C-1]):
IF DEN EQL Ø THEN GO TO L11:
Q1:= NUM/DEN:
GO TO L15;
L14: DEN:=(U[R,C]+U[R-2\SIGN(R)+1,C-1]+U[R+2\SIGN
          (M-R)-1,C-1]-3\U[R,C-1]);
IF DEN EQL Ø THEN GO TO L11;
Q1:= NUM/DEN:
GO TO L15:
L16: IF U[R,C-2] LEQ U[R,C-1] THEN GO TO L19:
DEN:=(U[R,C]+U[R,C-2]-2\U[R,C-1]);
IF DEN EQL Ø THEN GO TO L11:
Q1:= NUM/DEN:
GO TO L15:
L17: DEN:=(U[R,C]+U[R+2\SIGN(M-R)-1,C-1]-2\U[R,C-1]);
IF DEN EQL Ø THEN GO TO L11:
Q1:=NUM/DEN:
GO TO L15:
L18:DEN:=(U[R,C]+U[R-2\SIGN(R)+1,C-1]-2\U[R,C-1]);
IF DEN EQL Ø THEN GO TO L11:
Q1:= NUM/DEN:
GO TO L15:
L19:DEN:=(U[R,C]-U[R,C-1]):
IF DEN EQL Ø THEN GO TO L11:
Q1:= NUM/DEN:
GO TO L15:
L11: Q1:=Ø:
```

```
LI5:BEGIN
    REAL NUMER,DENOM;
   LABEL LD1,LD2,LD3,LD4,LD5,LD6,LD7,LD8;
IF U[R-2\SIGN(R)+1,C] LEQ Ø THEN GO TO LD1;
IF U[R,C] LEQ U[R-2\SIGN(R)+1,C] THEN GO TO LD1;
NUM:=Q[R-2\SIGN(R)+1,C]\(U[R,C]-U[R-2\SIGN(R)+1,C])\
          SQRT(U[R-2\SIGN(R)+1,C]/U[R,C]);
IF U[R-2\SIGN (R)+1,C-1] LEQ U[R-2\SIGN(R)+1,C] THEN
          GO TO LD2;
IF U[R-SIGN(R-1)+SIGN(1-R),C] LEQ U[R-2\SIGN(R)+1,C]
          THEN GO TO LD3;
IF U[R-2\SIGN(R)+1,C+1] LEQ U[R-2\SIGN(R)+1,C] THEN
          GO TO LD4;
DEN:=(U[R,C]+U[R-2\SIGN(R)+1,C-1]+U[R-SIGN(R-1)+
        SIGN(1-R),C]+U[R-2\SIGN(R),C+1]-4\U[R-2\SIGN
           (R)+1,C]);
IF DEN EQL Ø THEN GO TO LD1;
Q2:= NUM/DEN;
GO TO L24;
LD2:IF U[R-SIGN(R-1)+SIGN(1-R),C] LEQ U[R-2\SIGN(R)+1,
          C] THEN GO TO LD5;
IF U[R-2\SIGN(R)+1,C+1] LEQ U[R-2\SIGN(R)+1,C] THEN
          GO TO LD6;
DEN:=(U[R,C]+U[R-SIGN(R-1)+SIGN(1-R),C]+U[R-2\
     SIGN(R)+1,C+1]-3\U[R-2\SIGN(R)+1,C]);
IF DEN EQL Ø THEN GO TO LD1;
Q2:=NUM/DEN;
GO TO L24;
LD3:IF U[R-2\SIGN(R)+1,C+1] LEQ U[R-2\SIGN(R)+1,C]
          THEN GO TO LD7;
DEN:=(U[R,C]+U[R-2\SIGN(R)+1,C-1]+U[R-2\SIGN(R)+1,
        C+1]-3\U[R-2\SIGN(R)+1,C]);
IF DEN EQL Ø THEN GO TO LD1;
Q2:= NUM/DEN;
GO TO L24;
LD4:DEN:=(U[R,C]+U[R-2\SIGN(R)+1,C-1]+U[R-SIGN(R-1)
           +SIGN(1-R),C]-3\U[R-2\SIGN(R)+1,C]);
IF DEN EQL Ø THEN GO TO LD1;
Q2:= NUM/DEN;
GO TO L24;
LD5:IF U[R-2\SIGN (R)+1,C+1] LEQ U[R-2\SIGN(R)+1,C]
          THEN GO TO LD8;
DEN:=(U[R,C]+U[R-2\SIGN(R)+1,C+1]-2\U[R-2\SIGN
        (R)+1,C]);
IF DEN EQL Ø THEN GO TO LD1;
Q2:= NUM/DEN;
GO TO L24;
LD6:DEN:=(U[R,C]+U[R-SIGN(R-1)+SIGN(1-R),C]-2\
           U[R-2\SIGN(R)+1,C]);
IF DEN EQL Ø THEN GO TO LD1;
Q2:= NUM/DEN;
GO TO L24;
LD7:DEN:=(U[R,C]+U[R-2\SIGN(R)+1,C-1]-2\U[R-2\
          SIGN(R)+1,C]);
IF DEN EQL Ø THEN GO TO LD1;
Q2:= NUM/DEN;
GO TO L24;
LD8: DEN:=(U[R,C]-U[R-2\SIGN(R)+1,C]);
IF DEN EQL Ø THEN GO TO LD1;
Q2:= NUM/DEN;
GO TO L24;
LD1: Q2:=Ø;
  END;
```

```
L24:IF U[R,C+1] LEQ Ø THEN GO TO L29;
IF U[R,C] LEQ U[R,C+1] THEN GO TO L29;
NUM:=Q[R,C+1]\(U[R,C]-U[R,C+1])\SQRT(U[R,C+1]/U[R,C]);
IF U[R-2\SIGN(R)+1,C+1] LEQ U[R,C+1] THEN GO TO L3Ø;
IF U[R+2\SIGN(M-R)-1,C+1] LEQ U[R,C+1] THEN GO TO L31;
IF U[R,C+3\SIGN(N-1-C)-1] LEQ U[R,C+1] THEN GO TO L32;
DEN:=(U[R,C]+U[R-2\SIGN(R)+1,C+1]+U[R+2\SIGN(M-R)
     -1,C+1]+U[R,C+3\SIGN(N-1-C)-1]-4\U[R,C+1]);
IF DEN EQL Ø THEN GO TO L29;
Q3:= NUM/DEN;
GO TO L33;
L3Ø:IF U[R+2\SIGN(M-R)-1,C+1] LEQ U[R,C+1] THEN
           GO TO L34;
IF U[R,C+3\SIGN(N-1-C)-1] LEQ U[R,C+1] THEN GO TO L35;
DEN:=(U[R,C]+U[R+2\SIGN(M-R)-1,C+1]+U[R,C+3\
      SIGN(N-1-C)-1]-3\U[R,C+1]);
IF DEN EQL Ø THEN GO TO L29;
Q3:= NUM/DEN;
GO TO L33;
L31:IF U[R,C+3\SIGN(N-1-C)-1] LEQ U[R,C+1] THEN GO
           TO L36;
DEN:=(U[R,C]+U[R-2\SIGN(R)+1,C+1]+U[R,C+3\SIGN
      (N-1-C)-1]-3\U[R,C+1]);
IF DEN EQL Ø THEN GO TO L29;
Q3:= NUM/DEN;
GO TO L33;
L32:DEN:=(U[R,C]+U[R-2\SIGN (R)+1,C+1]+U[R+2\SIGN
           (M-R)-1,C+1]-3\U[R,C+1]);
IF DEN EQL Ø THEN GO TO L29;
Q3:= NUM/DEN;
GO TO L33;
L34:IF U[R,C+3\SIGN(N-1-C)-1] LEQ U[R,C+1] THEN
           GO TO L37;
DEN:=(U[R,C]+U[R,C+3\SIGN(N-1-C)-1]-2\U[R,C+1]);
IF DEN EQL Ø THEN GO TO L29;
Q3:= NUM/DEN;
GO TO L33;
L35:DEN:=(U[R,C]+U[R+2\SIGN(M-R)-1,C+1]-2\U[R,C+1]);
IF DEN EQL Ø THEN GO TO L29;
Q3:= NUM/DEN;
GO TO L33;
L36:DEN:=(U[R,C]+U[R-2\SIGN(R)+1,C+1]-2\U[R,C+1]);
IF DEN EQL Ø THEN GO TO L29;
Q3:= NUM/DEN;
GO TO L33;
L37:DEN:=(U[R,C]-U[R,C+1]);
IF DEN EQL Ø THEN GO TO L29;
Q3:= NUM/DEN;
GO TO L33;
L29: Q3:=Ø;
L33:IF U[R+2\SIGN (M-R)-1,C] LEQ Ø THEN GO TO L38;
IF U[R,C] LEQ U[R+2\SIGN(M-R)-1,C] THEN GO TO L38;
NUM:=Q[R+2\SIGN(M-R)-1,C]\(U[R,C]-U[R+2\SIGN(M-R)
        -1,C])\SQRT(U[R+2\SIGN(M-R)-1,C]/U[R,C]);
IF U[R+2\SIGN(M-R)-1,C+1] LEQ U[R+2\SIGN(M-R)-1,C]
           THEN GO TO L39;
IF U[R+2\SIGN(M-R)-1,C-1] LEQ U[R+2\SIGN(M-R)-1,C]
           THEN GO TO L4Ø;
IF U[R+SIGN(M-1-R)-SIGN(R+1-M),C] LEQ U[R+2\SIGN
           (M-R)-1,C] THEN GO TO L41;
```

```
DEN:=(U[R,C]+U[R+2\SIGN(M-R),C+1]+U[R+SIGN(M-1-R)
     -SIGN(R+1-M),C]-4\U[R+2\SIGN(M-R)-1,C]);
IF DEN EQL Ø THEN GO TO L38;
Q4:= NUM/DEN;
GO TO L42;
L39:IF U[R+2\SIGN(M-R)-1,C+1] LEQ U[R+2\SIGN(M-R)
          -1,C] THEN GO TO L43;
IF U[R+SIGN(M-1-R)-SIGN(R+1-M),C] LEQ U[R+2\SIGN(M-R)
          -1,C] THEN GO TO L44;
DEN:=(U[R,C]+U[R+2\SIGN(M-R)-1,C-1]+U[R+SIGN(M-1-R)
     -SIGN (R+1-M),C]-3\U[R+2\SIGN(M-R)-1,C]);
IF DEN EQL Ø THEN GO TO L38;
Q4:= NUM/DEN;
GO TO L42;
L40:IF U[R+SIGN(M-1-R)-SIGN(R+1-M),C] LEQ U[R+2\SIGN
          (M-R)-1,C] THEN GO TO L45;
DEN:=(U[R,C]+U[R+2\SIGN(M-R)-1,C+1]+U[R+SIGN(M-1-R)
       -SIGN(R+1-M),C]-3\U[R+2\SIGN(M-R)-1,C]);
IF DEN EQL Ø THEN GO TO L38;
Q4:= NUM/DEN;
GO TO L42;
L41:DEN:=(U[R,C]+U[R+2\SIGN(M-R)-1,C+1]+U[R+2\
        SIGN(M-R)-1,C-1]-3\U[R+2\SIGN(M-R)-1,C]);
IF DEN EQL Ø THEN GO TO L38;
Q4:= NUM/DEN;
GO TO L42;
L43:IF U[R+SIGN(M-1-R)-SIGN(R+1-M),C] LEQ U[R+2\SIGN
          (M-R)-1,C] THEN GO TO L46;
DEN:=(U[R,C]+U[R+SIGN(M-1-R)-SIGN(R+1-M),C]-2\U[
      R+2\SIGN(M-R)-1,C]);
IF DEN EQL Ø THEN GO TO L38;
Q4:= NUM/DEN;
GO TO L42;
L44:DEN:=(U[R,C]+U[R+2\SIGN(M-R)-1,C-1]-2\U[R+2\
          SIGN(M-R)-1,C]);
IF DEN EQL Ø THEN GO TO L38;
Q4:= NUM/DEN;
GO TO L42;
L45:DEN:=(U[R,C]+U[R+2\SIGN(M-R)-1,C+1]-2\U[R+2\
          SIGN(M-R)-1,C]);
IF DEN EQL Ø THEN GO TO L38;
Q4:=NUM/DEN;
GO TO L42;
L46:DEN:=(U[R,C]-U[R+2\SIGN(M-R)-1,C]);
IF DEN EQL Ø THEN GO TO L38;
Q4:= NUM/DEN;
GO TO L42;
L38:Q4:=Ø;
L42:Q[R,C]:=Q1+Q2+Q3+Q4;
L1Ø:END;
END;
```

```
FOR R:=0 STEP 1 UNTIL A,T+1 STEP T UNTIL M DO
  BEGIN
  IF U[R,2] LEQ 0 THEN GO TO L48;
  IF U[R,1] LEQ 0 THEN GO TO L48;
  GO TO IF U[R,1] GTR U[R,2] THEN L47 ELSE L48;
  L47:IF U[R+1,2] GTR U[R,2] THEN GO TO L61;
  IF U[R-2\SIGN(R)+1,2] GTR U[R,2] THEN GO TO L62;
  Q[R,1]:=Q[R,2]\SQRT(U[R,2]/U[R,1]);
  J[R]:=Q[R,1]\5.93@5\SQRT(U[R,1])/H*3;
  GO TO L52;
  L61:IF U[R-2\SIGN(R)+1,2] GTR U[R,2] THEN GO TO L63;
  Q[R,1]:=Q[R,2]\(U[R,1]-U[R,2])/(U[R,1]+U[R+1,2]-2\U
                [R,2])\SQRT(U[R,2]/U[R,1]);
     J[R]:=Q[R,1]\(5.93@5)\SQRT(U[R,1])/H*3;
  GO TO L52;
  L62:Q[R,1]:=(Q[R,2]\(U[R,1]-U[R,2])/(U[R,1]+U[R-2\SIGN(R)+1,2]-2
                  U[R,2]))\SQRT(U[R,2]/U[R,1]);
  J[R]:=Q[R,1]\5.93@5\SQRT(U[R,1])/H*3;
  GO TO L52;
  L63:Q[R,1]:=(Q[R,2]\(U[R,1]-U[R,2])/(U[R,1]+U[R+1,2]+U[
                 R-2\SIGN(R)+1,2]-3\U[R,2]))\SQRT(U[R,2]/U[R,1]);
  J[R]:=Q[R,1]\5.93@5\SQRT(U[R,1])/H*3;
  GO TO L52;
  L48:Q[R,1]:=0;
     J[R]:=0;
  L52: END;
FOR R:= A+1,V+1,D+1,F+1 DO
  BEGIN
  IF U[R,1] LEQ 0 THEN GO TO L57;
  IF U[R-1,1] LEQ 0 THEN GO TO L57;
  GO TO IF U[R-1,1] GTR U[R,1] THEN L58 ELSE L57;
  L58:QT:=Q[R,1]\SQRT(U[R,1]/U[R-1,1]);
  Q[R-1,1]:=Q[R-1,1]+QT;
  J[R-1]:=Q[R-1,1]\5.93@5\SQRT(U[R-1,1])/H*3;
  L57: END;
FOR R:=T,W,E,G DO
  BEGIN
  IF U[R,1] LEQ 0 THEN GO TO L53;
  IF U[R+1,1] LEQ 0 THEN GO TO L53;
  GO TO IF U[R+1,1] GTR U[R,1] THEN L54 ELSE L53;
  L54: QT:=Q[R,1]\SQRT(U[R,1]/U[R+1,1]);
  Q[R+1,1]:=Q[R+1,1]+QT;
  J[R+1]:=Q[R+1,1]\(5.93@5)\SQRT(U[R+1,1])/H*3;
  L53: END;
AV:=S/(M\N);
GO TO IF AV LEQ 0.000005 THEN L49 ELSE L1;
L49: IK:=IG:=IS:=IA:=0;
WRITE (TYPE,F12,ITER,AV);
WRITE(TYPE,F8);
FOR R:=B+1 STEP 1 UNTIL  P DO
```

```
    BEGIN
    WRITE (TYPE,F3,J[R]);
    IK:=IK+J[R]\H*2;
    END;
  WRITE (TYPE,F9);
  FOR R:=0 STEP 1 UNTIL A,T+1 STEP 1 UNTIL V DO
    BEGIN
    WRITE (TYPE,F3,J[R]);
    IG:=IG+J[R]\H*2;
    END;
  WRITE (TYPE,F10);
  FOR R:=W+1 STEP 1 UNTIL D DO
    BEGIN
    WRITE(TYPE,F3,J[R]);
    IS:=IS+J[R]\H*2;
    END;
  WRITE(TYPE,F11);
  FOR R:=E+1 STEP 1 UNTIL F DO
    BEGIN
    WRITE (TYPE,F3,J[R]);
    IA:=IA+J[R]\H*2;
    END;
   WRITE (TYPE,F4);
  WRITE (TYPE,F3,IK);
  WRITE(TYPE,F5);
  WRITE (TYPE,F3,IG);
  WRITE (TYPE,F6);
  WRITE (TYPE,F3,IS);
  WRITE (TYPE,F7);
  WRITE (TYPE,F3,IA);
  WRITE (TYPE,F13);
  WRITE (TYPE,F15,U[V,1],U[D,1],U[M,1]);
  IF WR0 EQL 0 THEN GO TO L51;
  WRITE (TYPE,F2,FOR R:=0 STEP 1 UNTIL M DO [FOR C:=0 STEP 1 UNTIL
            4 DO [U[R,C]]]);
  IF WR1 EQL 0 THEN GO TO L9;
  WRITE (TYPE,F2, FOR R:=0 STEP 1 UNTIL M DO[FOR C:=5 STEP 1
             UNTIL 9 DO[U[R,C]]]);
  L9:IF WR2 EQL 0 THEN GO TO L50;
  WRITE (TYPE,F14);
  WRITE (TYPE,F2,FOR R:=0 STEP 1 UNTIL M DO [FOR C:=1 STEP 1
            UNTIL 5 DO [Q[R,C]]]);
  L50: IF WR3 EQL 0 THEN GO TO L51;
  WRITE (TYPE,F2,FOR R:=0 STEP 1 UNTIL M DO[FOR C:=6 STEP 1
            UNTIL 10 DO [Q[R,C]]]);
  L51:$$A FINI
  END.

 END PUNCH 10.5 SEC.
```